[MATHS PROBLEM]

MULTIPLYING MAKES THINGS BIGGER

In everyday life, 'multiplying' usually means 'getting bigger', but in mathematics that isn't necessarily the case.

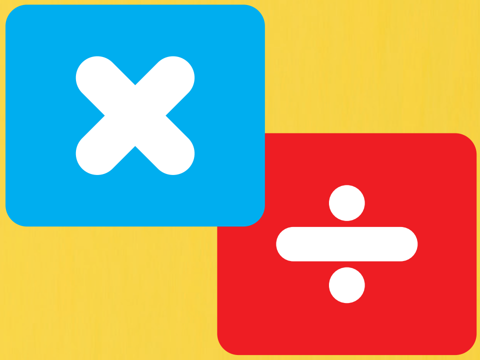
In this lesson, students examine different sizes of multipliers to see when multiplication or division results in a bigger or a smaller answer.

THE DIFFICULTY

This task is intended to bring to the surface possible confusion over multiplication:

Aka multiplied 8 by a certain number. He got an answer that was ***smaller than 8****. He thinks he must have made a mistake. What do you think?*

Students may agree that Aka must have made a mistake (e.g., he pressed 'divide' on the calculator instead of 'multiply'). Other students may realise that multiplication by a number that is ***less than 1*** will give an answer that is smaller than 8. They might think of specific examples, such as $\frac{1}{2}$ or 0 or -10.



THE SOLUTION

By thinking at the same time about multiplication and division, and multipliers that are greater than 1 and less than 1, students can develop a powerful sense of the expected size of answers to calculations.

Ajunni ***divided*** *8 by a certain number. She got an answer that was* ***larger than 8****. She thinks she must have made a mistake. What do you think?*

This may be a bit trickier for students to sort out. They may think, for example, that $8 \div \frac{1}{2} = 4$, rather than the correct answer of 16. However, if they record their calculations systematically, they may realise their error:

$8 \times 2 = 16$ so $16 \div 2 = 8$ and $16 \div 8 = 2$

$16 \times \frac{1}{2} = 8$ so $8 \div 16 = \frac{1}{2}$ and $8 \div \frac{1}{2} = 16$

Patterns like this can be powerful for convincing students things like the fact that $8 \div \frac{1}{2} = 16$, rather than 4. Everyone should agree with the blue statements. The red statements then follow by continuing the pattern. (Checking on a calculator may also reassure the very sceptical!)

Students could generate more examples and summarise their observations as:

8 × number greater than 1 = number **greater than 8**
8 × number less than 1 = number **less than 8**

8 ÷ number greater than 1 = number **less than 8**
8 ÷ number less than 1 = number **greater than 8**

The symmetry of these statements makes the pattern easy to recall, and, of course, the same thing will work not just for 8 but for any other positive number.

Students may benefit from lots of practice of this. For example, they could be asked to stand up if the answer is more than 15 and stay seated if the answer is less than 15. Then, you call out a series of calculations, like 15 ÷ 3.7 and 15 × 0.8, and the students need to quickly respond by standing up or staying seated. They don't need to try to calculate the exact answer – only to say if it must be greater than or less than 15.

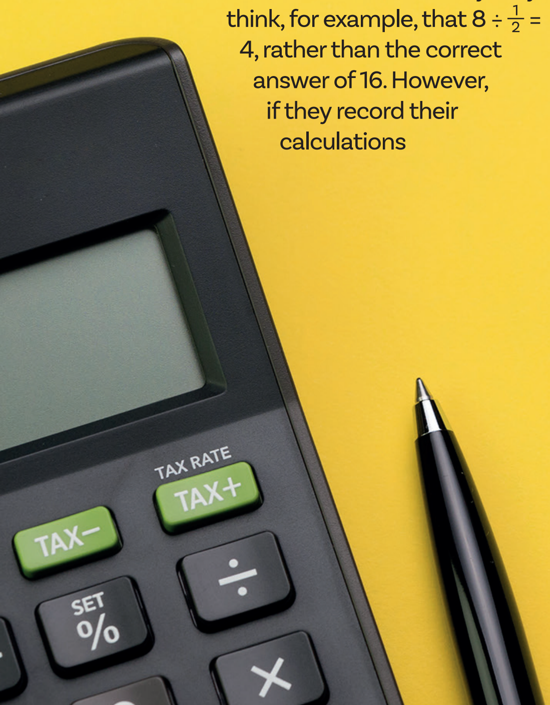
Checking for understanding

These final questions should give an indication of students' understanding of these ideas:

Choose the missing number that makes these calculations correct:

1. 15 ÷ ■ = 125 A. 120 B. 12 C. 1.2 D. 0.12
2. 15 × ■ = 1.8 A. 120 B. 12 C. 1.2 D. 0.12

D is the correct response in both cases.



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