

[MATHS PROBLEM]

NEGATIVE NUMBERS

Many students get confused when subtracting a negative number, says **Colin Foster**

In this lesson, students use positive and negative boxes to make sense of subtracting a negative number

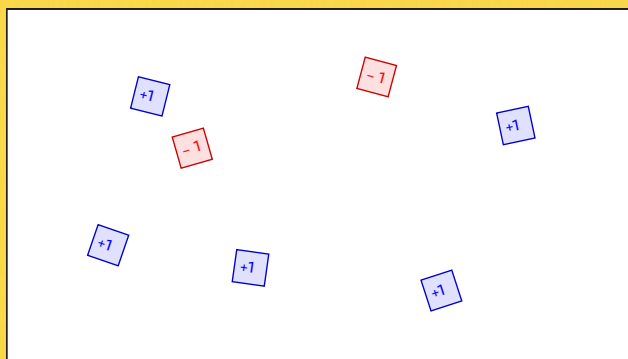
THE DIFFICULTY

Work out $(+3) + (-5)$, $(+3) - (-5)$, $(-3) + (-5)$ and $(-3) - (-5)$.

Students may struggle to answer all of these correctly, without getting muddled. The correct answers are -2 , $+8$, -8 and $+2$, respectively. Students may rely on rules such as “Two minuses make a plus”, but not know how to correctly apply them – perhaps thinking that $(-3) + (-5)$ should be equal to $+8$, rather than -8 .

THE SOLUTION

How much do I have here altogether?



Students might say 7, because there are 7 boxes. But five of the boxes are **positive** 1, and two of them are **negative** 1, so the total value is $+3$. The key idea is that one $+1$ and one -1 cancel each other out. Students might benefit from cutting out squares of paper with $+1$ written on some and -1 written on others. Then they can pair up a $+1$ and a -1 to make a ‘zero pair’.

Show me another way to make the same total. And another way. And another.

Students will realise they can add or subtract as many ‘zero pairs’ as they like, and the total will remain $+3$.

Which of our four calculations are easy to do with the boxes, and which are trickier?

The additions are not too difficult. $(+3) + (-5)$ means starting with three $+1$ s and five -1 s. This gives 3 zero pairs, with two -1 s left over, so the answer is -2 . $(-3) + (-5)$ means beginning with three -1 s and getting five more -1 s, taking us to -8 .

The subtractions are trickier, because in both cases we are taking away more -1 s than we have. To do $(+3) - (-5)$, starting with three $+1$ s, we need to add (at least) five zero pairs, so we have eight $+1$ s and five -1 s. **Now**, we can take away -5 , to leave $+8$.

Similarly, with $(-3) - (-5)$, starting with three -1 s, we need to add (at least) two more ‘zero pairs’, so we have enough -1 s to subtract. This means we have two $+1$ s and five -1 s. **Now**, we can take away -5 , to leave $+2$.

Checking for understanding

Make up four directed number additions and subtractions, like those I gave you at the start. For each one, give the correct answer, with an explanation why, and an incorrect answer someone might be likely to give, saying why they might think it.

For an animated version of this way of thinking, see <https://www.twominutemath.org/gallery/addsubtractnegnumswork.html>



Colin Foster (@colinfoster77) is a Professor of Mathematics Education in the Department of Mathematics Education at Loughborough University. He has written many books and articles for mathematics teachers. foster77.co.uk, twominutemath.org