[MATHS PROBLEM]

PERIMETERS OF SECTORS

Colin Foster offers some advice for preventing confusion when your students are tasked with finding the perimeters of sectors

In this lesson, students are asked to come up with as many examples as they can of sectors with a perimeter of 24 cm.

THE DIFFICULTY

Can you calculate the perimeter of the sector in Fig 1?

Students may calculate just the arc length (13.96 cm), failing to include the two radii, or they may calculate the ***area*** instead (34.90 cm^2), possibly mistakenly adding twice the radius to ***that***.

The length of an arc of radius r and angle θ degrees is $\frac{\theta}{360} \times 2\pi r$, and the perimeter is the sum of the red arc length and the two blue radii shown in Fig 2 – so for this sector, the perimeter comes to

$2 \times 5 + \frac{160}{360} \times 2\pi \times 5$, which is equal to 23.96 cm (correct to 2 decimal places), or about 24 cm.

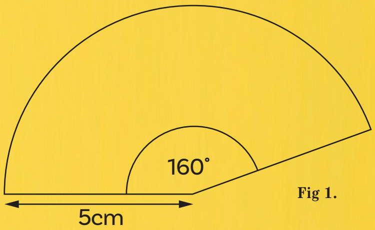

Fig 1.

Fig 2.

THE SOLUTION

Invent some other sectors with a perimeter of as close to 24 cm as you can.

This is not as straightforward as it might seem, since changing the radius changes both the length of the two straight line segments and the length of the arc at the same time. Students will probably work by trial and error, which will generate lots of useful practice. Some students might take an algebraic approach. If we write a formula for the perimeter P of a sector,

$$P = 2r + \frac{\pi r\theta}{180}$$

then we can rearrange this to make θ the subject:

$$\theta = \frac{180\,(P - 2r)}{\pi r}$$

This enables us to calculate the required value of θ for any value of r we choose. All the possible cases for integer r are shown in the table, and students could be asked to draw a graph of θ against r, as shown in Fig 3. (Students may need help seeing why the cases r = 1 and 2 do **not** give possible values of θ, and might argue about the r = 12 case!)

r (cm)	θ (°) correct to 1 decimal place
3	343.8
4	229.2
5	160.4
6	114.6
7	81.9
8	57.3
9	38.2
10	22.9
11	10.4

You could also ask students to explore the ***areas*** of these sectors, and to find the sector with perimeter 24 that has the ***largest*** area. It is possible to express the area A in terms of the perimeter and radius as $A = \frac{r(P-2r)}{2}$.

Completing the square, $A = \frac{p^2}{16} - (r - \frac{p}{4})^2$,

meaning that A is a maximum when $r = \frac{p}{4}$,

which is when r = 6 cm and A= 36 cm^2.

Checking for understanding

To assess students' understanding, you could ask them to invent one sector with a ***perimeter of 12 cm*** and another sector with ***an area of 12 cm***^{2}.

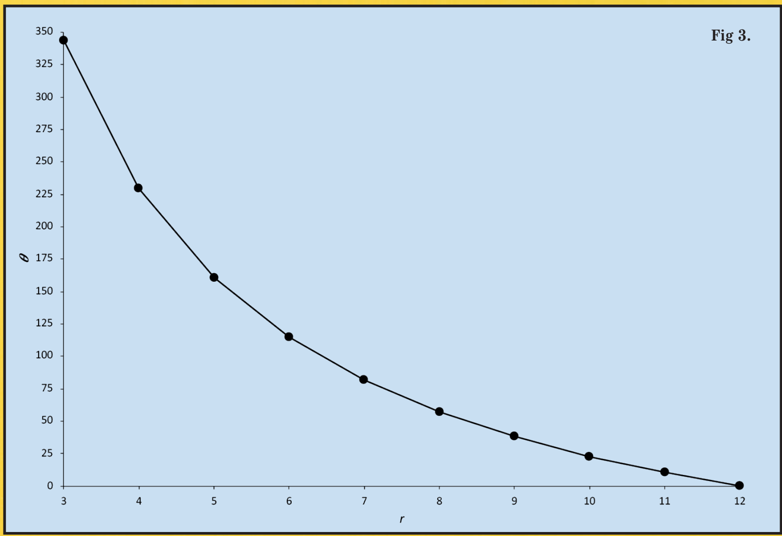

Fig 3.

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