[MATHS PROBLEM]

PERPENDICULAR GRADIENTS

Colin Foster unpacks why relationships between the gradients of perpendicular lines can cause students some confusion

In this lesson, students draw lines with different gradients to see what is needed for the lines to be perpendicular.

THE DIFFICULTY

I'm thinking of a line that has a gradient of $\frac{2}{3}$
What is the gradient of a line that is ***perpendicular*** *to this line?*
A. $\frac{2}{3}$ *B.* $-\frac{2}{3}$ *C.* $\frac{3}{2}$ *D.* $-\frac{3}{2}$

Students may need to remind each other that 'perpendicular' means 'at right angles'. If they have no idea about the correct answer, that is OK, because that is the point of today's lesson.

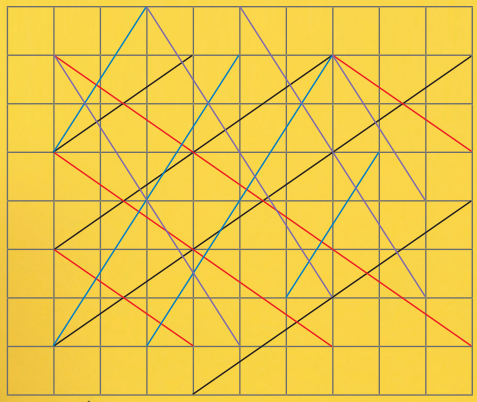
THE SOLUTION

What does a line with gradient $\frac{2}{3}$ *look like?*

Students could answer on mini-whiteboards. (This would be easier if the mini-whiteboards had a squared grid background.) If they have all used, say, black ink, then you could next ask:

Now, using red pen, draw a line with gradient $-\frac{2}{3}$
Now, using blue pen, draw a line with gradient $\frac{3}{2}$
Now, using purple pen, draw a line with gradient $-\frac{3}{2}$

Several possible examples of correct lines are shown below.



Which colour line is ***perpendicular*** *to the original black line?*

The purple lines, with gradient $-\frac{3}{2}$, are perpendicular to the black lines.

Students may notice that the red and blue lines are ***also*** perpendicular to each other.

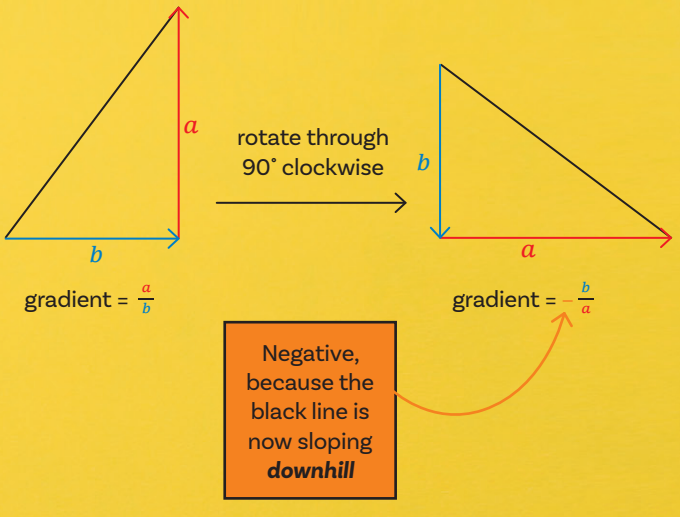
In general, lines with ***gradients that multiply to make -1*** are perpendicular to each other:

$$\left(\frac{2}{3}\right) \times \left(-\frac{3}{2}\right) = -1$$

$$\left(-\frac{2}{3}\right) \times \left(\frac{3}{2}\right) = -1$$

The converse is ***almost*** true, except for the case of horizontal and vertical lines, which ***are*** perpendicular, but do ***not*** have gradients with a product of -1.

Checking for understanding
To assess students' understanding, ask them to try to prove this result for the general case of a line with gradient $\frac{a}{b}$, where $a, b \neq 0$. For example, they could make an annotated drawing something like this:



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