[ MATHS PROBLEM ]

# PROPORTIONALITY

Proportionality is a notoriously difficult idea for students, observes Colin Foster...

**In this lesson, students see how the speed at which they listen to a podcast affects how much time they save.**

## THE DIFFICULTY

*Yesterday, I listened to a podcast at $1\frac{1}{2}$ speed, rather than at normal speed.*
*It saved me 15 minutes.*
*What can you work out from this?*

This is a hard question that students will likely struggle to answer. Part of the difficulty is that the question doesn't even tell you what it wants you to work out – only that ***something*** can be deduced!

If your students are completely stuck, then it's fine to leave this question hanging, because by the end of the lesson they ***will*** be able to answer it.

## THE SOLUTION

*What does '$1\frac{1}{2}$ speed' actually mean?*

If this is too hard for students to express, try this instead:

*What does 'double speed' mean?*

This is easier. It means the podcast will take ***half as long*** to play as it would at normal speed.

*Give me some more examples like this.*

Students will offer things like '4 times the speed means it takes a quarter of the time' – although at 4 times the speed, it might be impossible to follow what anyone on the podcast is actually saying!


Students could make a table with some examples, like this:

| *speed* | *time reduction* |
|---|---|
| 1 | 0 |
| 1.5 | ? |
| 2 | $\frac{1}{2}$ |
| 3 | $\frac{2}{3}$ |
| 4 | $\frac{3}{4}$ |
| 5 | $\frac{4}{5}$ |

Note that if the podcast runs 4 times as fast, it takes $\frac{1}{4}$ of the time. That's a reduction in time spent of $\frac{3}{4}$, not $\frac{1}{4}$.

Clearly the ? in the table needs to be a fraction between 0 and $\frac{1}{2}$. This is a good way to think about reciprocals. The time it takes to listen to a $n$-speed podcast is $\frac{1}{n}$ of the normal speed. So, the time it takes to listen to an $\frac{n}{m}$ speed podcast is going to be $\frac{m}{n}$ of the normal speed.

This tells us that for a $1.5 = \frac{3}{2}$ speed podcast, the listening time will be $\frac{2}{3}$ of the normal speed, so the reduction will be $\frac{1}{3}$. Don't rush through this – let students create lots of examples to see why it works.

With a $\frac{1}{3}$ reduction corresponding to 15 minutes, in the original question, the original podcast time, at normal speed, must have been three times as much, so it was a 45-minute podcast. That's the thing we can deduce.

### Checking for understanding

*Can you invent a similar problem to this one that has a nice neat answer, like this one did?*

Students will need to think of a speed that has a 'nice' reciprocal in order to make this work.

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