[MATHS PROBLEM]

RANDOM-WALK PLOTS

Understanding patterns within randomness can be challenging, says **Colin Foster**

In this lesson, students make a random-walk plot to make sense of the outcomes from flipping a coin

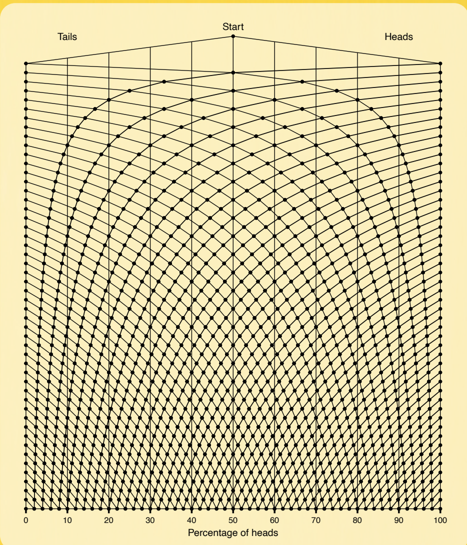
THE DIFFICULTY

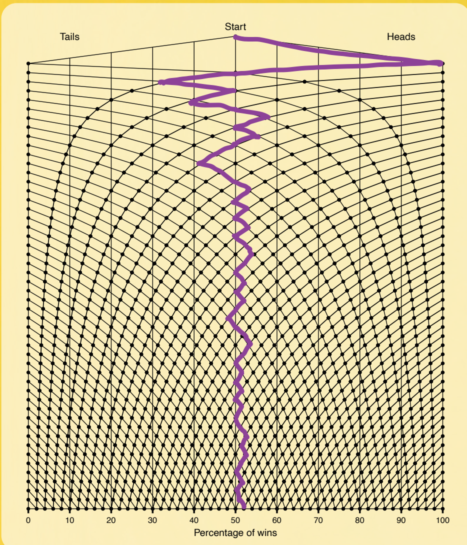
If you throw an ordinary coin, can you predict whether it will come up heads or tails?

Students will probably say no, because the coin could show either heads or tails, and if it's a fair coin, we can't know in advance which. The outcome is random, depending on chance, meaning we can't predict which side it will be.

*If you throw **a hundred** ordinary coins, can you predict what will happen?*

This time, we can predict that there should be about 50 heads and 50 tails. It's quite mysterious that we ***can't*** predict the outcome of ***any one*** of the 100 coins. But we can predict the outcome of the entire group! Students often find this aspect of randomness puzzling.





THE SOLUTION

*Instead of throwing 100 coins, we're going to throw **one coin 100 times**, so we can see the pattern developing gradually. Each time we throw the coin, we won't know if it will come up heads or tails – we'll assume they're equally likely outcomes. But nevertheless, we'll see a pattern develop.*

Give students a copy of a blank ***random-walk plot***, pictured above left (available from tinyurl.com/ts152-MP1). Beginning at the top ('Start'), they proceed to throw a coin 50 times, drawing one step right or one step left with each throw, depending on whether the coin comes up heads or tails. The sequence 'HTTHT HHTHT TTHHH THTHT HHTTH THTTH HHTTH THTHT HHTHT THTHH' is shown in the completed example on the right.

The plot is designed so that at every stage, the position you've reached along the scale at the bottom tells you the percentage of heads obtained up to that point. (The design is like a fraction wall.)

When they have finished, students can do a 'gallery walk' to see everyone else's plots. Even though they may look quite different at the top, they should all be quite similar nearer the bottom.

Checking for understanding

What do you see? What does it mean?

Students will realise that although they can't predict the outcome on any one throw, the ***percentage*** of heads gets very close to 50% after a large number of throws. This is known as the ***law of large numbers***. Indeed, it's ***because*** the outcome of an individual throw is completely unpredictable that we get the predictable result with large numbers of throws!

Colin Foster (**@colinfoster77**) is a Professor of Mathematics Education in the Department of Mathematics Education at Loughborough University. He has written many books and articles for mathematics teachers. **foster77.co.uk**, **blog.foster77.co.uk**