[MATHS PROBLEM]

RECURRING DECIMALS

Students are often confused about recurring decimals and their relationships to fractions

In this lesson, students compare terminating and recurring decimals and the fractions equivalent to them

THE DIFFICULTY

Look at these quantities. Do any of them have the same value? Put them in order of size, from smallest to largest.

$\frac{1}{3}$ 0.33 $\frac{3}{10}$ $\frac{33}{99}$ $\frac{0.3}{1}$ 0.3 $\frac{3}{9}$ $0.\dot{3}$ $\frac{33}{100}$

Students may think that $\frac{1}{3}$ and 0.3 or 0.33 are the same, but although this is ***approximately*** true it is not exactly true.

There are actually three different numbers in this list, and the correct order is:

$$\frac{3}{10} = \frac{0.3}{1} = 0.3 \quad < \quad \frac{33}{100} = 0.33 \quad < \quad \frac{1}{3} = \frac{3}{9} = \frac{33}{99} = 0.\dot{3}$$

Students might notice that 0.3 and 0.33 are **terminating** decimals, and $0.\dot{3}$ is **recurring**, but all three numbers are **rational**, because each can be expressed as a fraction containing only integers. This contrasts with numbers such as $\sqrt{3}$ or 4π, which are **irrational**, and ***can't*** be expressed as fractions of integers.

THE SOLUTION

Using written methods or calculators, convert each of these fractions into a decimal.
Say whether the decimal is terminating or recurring.
Describe what you notice about your answers.
(Answers are given in red below.)

Fraction	*Decimal*	*Terminating or recurring?*
$\frac{7}{10}$	0.7	T
$\frac{7}{9}$	$0.\dot{7}$	R
$\frac{7}{100}$	0.07	T
$\frac{7}{99}$	$0.\dot{0}\dot{7}$	R
$\frac{37}{100}$	0.37	T
$\frac{37}{99}$	$0.\dot{3}\dot{7}$	R
$\frac{37}{1000}$	0.037	T
$\frac{37}{999}$	$0.\dot{0}3\dot{7}$	R

As students complete this task, they will be confronted with the difference between something like 0.7 and $0.\dot{7}$, forcing them ***not*** to treat them as identical. They may conjecture (correctly) that $0.\dot{p} = \frac{p}{9}$ and $0.\dot{p}\dot{q} = \frac{pq}{99}$, etc., for any digits p and q. They may also notice that any integer fraction with 10, 100, ... in the denominator gives a terminating decimal. But it is ***not*** the case that any fraction with 9, 99, ... in the denominator gives a recurring decimal, and students might try to find counter examples (e.g., $\frac{990}{99}$).

Checking for understanding

Write each of these decimal numbers as a simplified fraction:

$0.\dot{8}$ 0.8 $0.\dot{0}\dot{8}$ $0.\dot{3}\dot{8}$ 0.38 $0.\dot{8}\dot{3}$ $0.\dot{8}0\dot{3}$ 0.803 $0.0\dot{8}$ $0.3\dot{8}$

Put your answers in order of size, from smallest to largest.

The answers are:

$\frac{8}{9}$ $\frac{4}{5}$ $\frac{8}{99}$ $\frac{38}{99}$ $\frac{19}{50}$ $\frac{83}{99}$ $\frac{803}{999}$ $\frac{803}{1000}$ $\frac{8}{90}$ $\frac{7}{18}$

and in order of size they are:

$\frac{8}{99}$ $\frac{8}{90}$ $\frac{19}{50}$ $\frac{38}{99}$ $\frac{7}{18}$ $\frac{4}{5}$ $\frac{803}{1000}$ $\frac{803}{999}$ $\frac{83}{99}$ $\frac{8}{9}$

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