Lesson plan: MATHS KS4

ROBOT RENDEZVOUS

Distance-time graphs are a powerful approach to solving many problems relating to motion, says **Colin Foster**

In this lesson, students explore a scenario involving two robots, each moving at a (different) constant speed along the same straight line. Students draw distance-time graphs to describe the movement of the robots and predict when and where they will meet. Increasingly complicated scenarios can all be modelled with the same graphical approach, allowing students to see the power of graphical representations to make sense of situations.

Q When and where will two robots moving at constant speed along the same straight line meet?

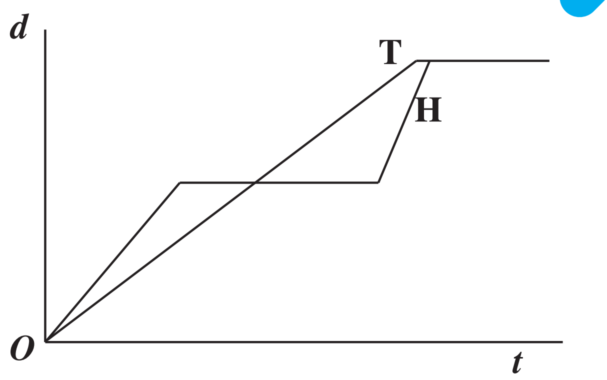
STARTER ACTIVITY

Q *Do you know the story of the hare and the tortoise?*

A student who knows the story could briefly recount it.

Q *I would like you to tell this story using a distance-time graph.*

Students could do this on mini-whiteboards. You may need to clarify that there will be two lines on the same set of axes – one for the tortoise and one for the hare. Students should check each other's graphs, being careful about details such as that the hare, when running, always has a steeper gradient than the tortoise and that the tortoise wins, but only just. A possible graph is:



WHY TEACH THIS?

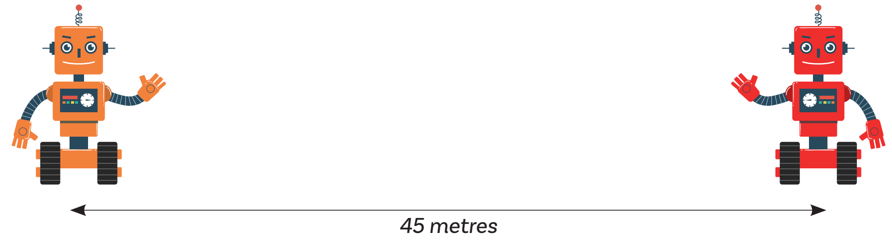
Many real-life problems involve speed, distance and time. Representing motion using distance-time graphs can be very helpful in making sense of complicated situations.

KEY CURRICULUM LINKS

+ Interpret distance-time graphs
+ Plot and interpret graphs of non-standard functions in real contexts

MAIN ACTIVITY

Q Imagine two robots, both moving at steady speeds along the same straight line.
The orange robot travels at 5 metres per second.
The red robot travels at 10 metres per second.
They start off facing each other 45 metres apart in a room 45 metres long.
Whenever they bump into each other, or a wall, they begin to move in the opposite direction, at the same speed.



Q Can you describe in words what will happen?

They should try to give as much detail as possible. Students could act out the movements at the front of the classroom, perhaps slowing down the timings and stepping the movements as someone else counts the 'seconds'.

Q At what times and distances will the robots meet? Why?

Students might approach this algebraically. Taking the origin at the left-hand wall, behind the orange robot's starting position, we can write down a pair of equations, where d are the distances in metres and t is the time in seconds:

$$d_O = 5t$$
$$d_R = 45 - 10t$$

These will be valid until the robots meet, but then new equations will be needed, so an algebraic approach may be difficult.

The starter should suggest that a graphical solution might be possible. Students might calculate distances at different times and make a table of numbers to get a sense of what is going on and then plot these on a graph, or they might draw directly onto the graph as they think about what is happening.

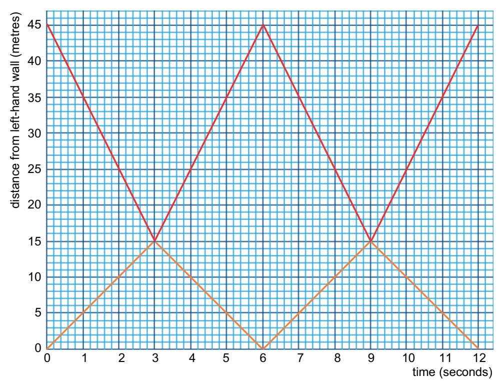
If students are stuck, questions like 'Where will the robot be after 1 second?' and 'Are the robots getting closer together or further apart now?' may help.

When students have worked out what is happening, they should modify the scenario, changing the speeds and the length of the room, and also the rules, such as that the robots ***pass*** each other, rather than bounce off each other, when they meet. What difference does this make to the graphs and to what happens?

DISCUSSION

Discuss what students have worked out and how they went about it.

Q. *Which problems did you work on? How did you work out the graphs? How did you use the graphs to decide what was happening? What was difficult or confusing? In what ways were the graphs useful? What other kinds of problems could distance-time graphs help to solve? What was hard about using graphs to solve this problem?*



Students could share the graphs that they drew, perhaps using a visualiser. For example, the original scenario could be represented by the graph below, where the colours of the lines match those of the robots. We can see that the robots repeat their motion on a period of 6 seconds. They always meet 15 metres from the left-hand wall, every 6 seconds starting 3 seconds after the beginning.

A graph is even more useful for examining more complicated movements, such as when the robots ***pass***, rather than bounce off each other. There should be much for students to talk about and work out.

ADDITIONAL RESOURCE

A related task is available at **nrich.maths.org/4808**

GOING DEEPER

Confident students could imagine the two robots moving around in a circle. When and where does one robot overtake the other? How do the graphs need to be modified for this change?

THE AUTHOR

Colin Foster is an Associate Professor in the School of Education at the University of Leicester. He has written many books and articles for mathematics teachers (see **www.foster77.co.uk** and **@colinfoster77** on Twitter).