WHY TEACH THIS?

Despite what people sometimes think, mathematics is not just about numbers – it is about many other things, such as shapes. Mathematics provides a powerful language for describing and reasoning about geometrical shapes. Identifying shapes based on their properties and using precise language to describe them allows us to reason mathematically.

SHAPING UP

THE WORLD AROUND US IS TEEMING WITH POLYGONS; AND MATHEMATICS GIVES US A POWERFUL LANGUAGE FOR DESCRIBING THEIR PROPERTIES, SAYS COLIN FOSTER...

Students sometimes think that being good at mathematics is simply about being adept with numbers – things like knowing their tables off by heart. While numbers certainly play a large part in school mathematics, there is a great deal of the subject that has very little to do with them. A good example is school geometry – the study of shapes. Learning about polygons can often be difficult because of the many different names (and awkward spellings) involved. However, knowing the different properties of the various polygons enables us to identify, classify and describe them. It gives us a language to talk and reason about them. So deciding whether a shape is, say, a parallelogram or not entails examining the definition of a parallelogram on the one hand and the properties of the shape on the other. In this lesson, students use just seven points on a coordinate grid as the vertices of a multitude of shapes. Which can they find and what are their properties? How can they be sure if something that looks like a rhombus really is one or not?

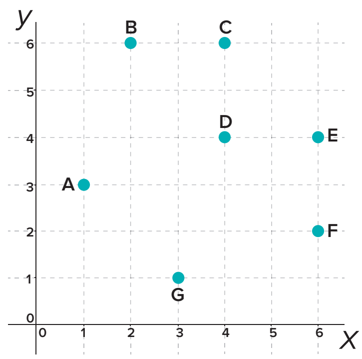
STARTER ACTIVITY

Put this list of points on the board:

A(1, 3), B(2, 6), C(4, 6), D(4, 4), E(6, 4), F(6, 2) and G(3, 1).

Q. Draw axes from 0 to 6 in both directions and plot and label these points. Do it on your own and then check to see if you have the same picture as the person next to you.

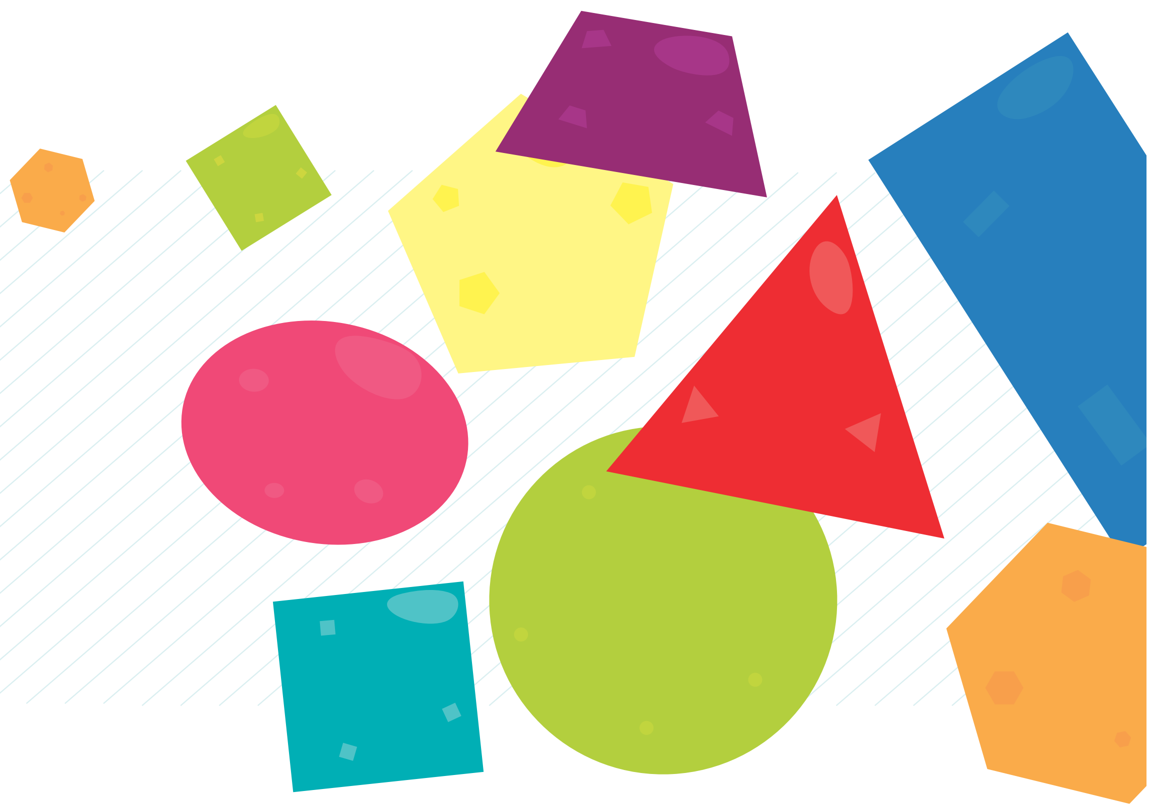
It is important that students have this correct, otherwise their results will not match up later! They must use the same scale on both axes. If students are using the same scale as each other (e.g., on centimetre-squared paper) then they could place their answer on top of their neighbour's to check that they are the same. You can see from the coordinates that the points should be symmetrical about the line $y = x$, which is another way to check.



A PRINTABLE VERSION OF THIS DIAGRAM IS AVAILABLE AT teachsecondary.com/downloads/maths-resources

Q. Look at your grids. Suppose I say that BCD is a right-angled triangle. Do you agree? What else can you say about it?

Students will pick up the idea that you are running letters together to identify a polygon, if they have not met this convention before. You could ask students to say which vertex is the right angle (C) and which side is the hypotenuse (BD), if they are familiar with that term. They might note that BCD is an *isosceles* right-angled triangle, since BC = CD.



Q. Can someone find another right-angled triangle? What else can you say about it?

Students will probably spot CDE and DEF and might realise that these are not only also isosceles right-angled triangles but that they are ***congruent*** to BCD (i.e., exactly the same shape and size). Someone might offer DCB or some other permutation of BCD – in some situations we might regard this as a different triangle from BCD, but for this lesson we won't count that as another triangle.

Q. Can you find **another** *right-angled triangle?*

Once CDE and DEF are taken, this should be harder, and it might be good to allow students to think about this in pairs. They may think that triangles such as ABD are right-angled and might need to measure with a protractor, or count the squares, to realise that ABD is actually an acute-angled triangle. There are four further right-angled triangles: ACE, CEG, AEG and ACG. Can students find them all, and justify that there can be no others?

Q. What other triangles can you find and name?

Students should be able to find acute-angled isosceles triangles and acute-angled and obtuse-angled scalene triangles, but not obtuse-angled isosceles triangles or equilateral triangles.

MAIN ACTIVITY

Q. What do we mean by a quadrilateral?

Students should know that this is any four-sided polygon.

Q. I want you to find as many different kinds of quadrilaterals as you can in the diagram and write down for each one the four letters and the name.

Students need to know that for a shape with more than three vertices we write the letters ***cyclically***, so for the parallelogram ABDG we could instead write BDGA but not ABGD, because BG is a diagonal, not a side.

If students are very rusty over the names of the different kinds of quadrilaterals, you could briefly brainstorm them on the board, including their spellings, or allow this to emerge during the lesson.

Students should be able to find parallelograms and trapeziums and kites without too much difficulty, although they may sometimes need to rotate their paper, or their head, to recognise shapes in non-standard orientations. The rectangle ACEG is often a surprise, although finding the four right-angled triangles earlier may suggest it. Many quadrilaterals can be given several different names. For instance, all rectangles are also parallelograms, since they have two pairs of parallel sides. If we define a trapezium as a quadrilateral with ***at least*** one pair of parallel sides, then all parallelograms are also trapeziums.

Some students may think that ABDG is a kite or a rhombus (which they may call a "diamond"), but in fact no pairs of adjacent sides are equal, so it is only a parallelogram. Students may want to call ACDE an arrowhead, but AC ≠ AE, so it is just a concave quadrilateral, as it is not symmetrical.

You could ask students who finish early to say what kinds of quadrilateral they ***can't*** find (e.g., square, rhombus, arrowhead). If they were allowed to add one new point H of their choice, where would they put it, so as to be able to make as many new kinds of quadrilateral as possible? (The point H(2, 4) would be a possible choice, creating a square BCDH, which is also a rhombus, and an arrowhead ABHD.) Or students could consider moving one point to allow them to make as many kinds of quadrilateral as possible.

+ KEY RESOURCE

DON'T MISS THIS...

Integral covers A level Mathematics and Further Mathematics for AQA, Edexcel, OCR, OCR(MEI) and WJEC. As well as notes, exercises, online tests and interactive resources, Integral contains many resources to facilitate classroom discussion and group work.

Examples of these include jigsaws of triangles containing mathematical expressions to match up and sets of cards showing mathematical objects to sort by shared properties. Activities like this encourage students to talk, learn from one another and build collaborative relationships.

Integral is developed by Mathematics in Education and Industry, an independent charity committed to improving maths education.

Visit integralmaths.org for samples.

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is an Assistant Professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

ADDITIONAL RESOURCES

THERE IS AN INTERESTING ARTICLE AT NRICH.MATHS.ORG/10674 ON WAYS OF USING ***GEOBOARDS***, WHICH ARE BOARDS WITH PINS STICKING OUT IN ROWS – ELASTIC BANDS CAN BE STRETCHED AROUND THEM TO MAKE POLYGONS. THERE IS A NICE VIRTUAL GEOBOARD IS AVAILABLE AT NRICH.MATHS.ORG/ARTGALLERY/#/.

STRETCH THEM FURTHER

STUDENTS COULD CHOOSE THEIR OWN SET OF SIX PAIRS OF COORDINATES. CAN THEY FIND A SET OF SIX PAIRS OF COORDINATES THAT ALLOWS ***ALL*** THE DIFFERENT KINDS OF QUADRILATERALS TO BE MADE? OR A SET OF SIX PAIRS OF COORDINATES THAT ALLOWS ***ONLY ONE*** KIND OF QUADRILATERAL TO BE MADE? WHAT OTHER QUESTIONS CAN THEY ASK AND ANSWER?

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about which quadrilaterals they have found and which they have not. Which were the hardest ones to find? Are there any they are unsure about? How do they justify the names that they have given their shapes? What modifications to the points did they consider in order to generate more different quadrilaterals?

The table below shows shapes that can be found.

Quadrilateral	kite	parallelogram	rectangle	trapezium
BCEF, ABFG				✓
BCED, ADFG, ABDG, CEFD		✓		✓
BCDA, DEFG	✓			
ACEG		✓	✓	✓