[MATHS PROBLEM]

SIMULTANEOUS EQUATIONS

When using the elimination method to solve simultaneous equations, students are often unsure whether to add or subtract.

In this lesson, students explore what happens when they add and subtract pairs of simultaneous equations.

THE DIFFICULTY

Look at the below four pairs of simultaneous equations.

Which ones could you solve by adding the equations together?
Which ones could you solve by subtracting one equation from the other?
Students may be unsure and not know how to decide.

$5x + 2y = 17$ $4x - 2y = 10$	$5x + y = 16$ $4x + y = 13$	$3x + 4y = 13$ $3x - 5y = 4$	$5x - 3y = 12$ $2x - 3y = 3$

THE SOLUTION

*Don't worry about **solving** the equations just yet. All I want you to do is simply add together each pair of equations. And also subtract each pair of equations. See what you get.*

Students should obtain the following:
Adding...

$9x + 0y = 27$	$9x + 2y = 29$	$6x - y = 17$	$7x - 6y = 15$

Subtracting the second equation from the first equation...

$x + 4y = 7$	$x + 0y = 3$	$0x + 9y = 9$	$3x + 0y = 9$

Students may not bother to write the $0x$ and $0y$ where there are no x and y terms, and this is fine. They may make errors, particularly when subtracting the negative terms, so, for the subtractions, they may end up with the wrong answers in red below:

$x + 0y = 7$		$0x - 9y = 9$	$3x - 6y = 9$

> *"It can help if students remember the following: When the Signs are the Same you Subtract"*

Writing out the difficult subtractions explicitly may help:

$2y - (-2y) = 4y$	$4y - (-5y) = 9y$	$-3y - (-3y) = 0y$

When does adding eliminate an unknown?
This happens when two terms are equal in magnitude, but of opposite sign (e.g., $2y$ and $-2y$).

When does subtracting eliminate an unknown?
This happens when two terms are equal in magnitude, and of the same sign (e.g., $-3y$ and $-3y$).

Sometimes, it can help if students remember the following: *When the **S**igns are the **S**ame you **S**ubtract.*

*Can you find a pair of equations where **either** adding **or** subtracting will lead to elimination of one of the unknowns?*

An example would be $3x + 2y = 11$ and $3x - 2y = 7$.

The solution to all of these pairs of equations is $x = 3$, $y = 1$.

Checking for understanding

To assess students' understanding, ask them to create four pairs of simultaneous equations of their own, two of which can be solved by adding the equations, and two of which can be solved by subtracting the equations. They should label clearly which are which.

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