[MATHS PROBLEM]

SOLVING SIMULTANEOUS EQUATIONS

Setting up and solving simultaneous equations from a context is often challenging, says **Colin Foster**

In this lesson, students use a Fibonacci-like sequence as context for simultaneous equations

THE DIFFICULTY

*From the third box onwards, the number in each box below is the sum of the numbers in the **two** previous boxes.*

1	2	3	5	8	13

Can you find the missing numbers here?

5		9			

The second box must be $9-5=4$, and it then follows that the three final boxes must be 13, 22 and 35.

Now try this one:

		16			62

Although, again, two numbers are given, this one is much harder, because we don't know either of the two starting numbers (only their sum). If no one can do it, this nicely sets up the motivation for the main activity.

THE SOLUTION

These box puzzles are very easy to make up, by choosing the first two numbers and doing the additions. But they can be hard to solve, and involve a lot of trial and improvement. Alternatively, we can use algebra to make it much easier.

We will let a and b represent the first two numbers (whether known or not).

*Can you find the missing **expressions** here?*

a	b				

The complete sequence is

a	b	$a+b$	$a+2b$	$2a+3b$	$3a+5b$

Learners will notice that the coefficients match the numbers that were in the first set of boxes presented at the start, and that these are called the ***Fibonacci numbers*** ((1), 1, 2, 3, 5, 8, 13, 21...).

How can you use these expressions to solve the difficult box puzzle above?

Learners will see that $a+b$ must be equal to 16 and $3a+5b$ must be equal to 62. Now learners can practise solving these equations simultaneously by elimination:

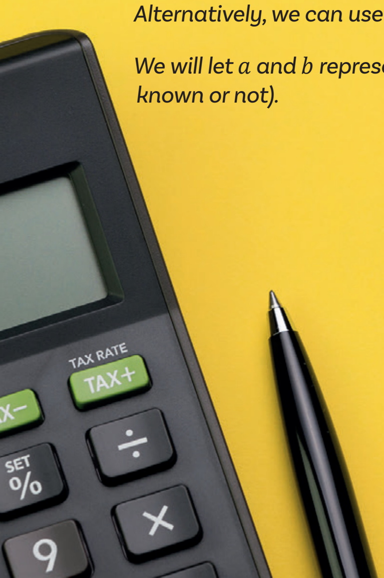
$$\begin{aligned}
a+b &= 16 && (1)\\
3a+5b &= 62 && (2)\\
3a+3b &= 48 && (3) = 3\times(1)\\
2b &= 14 && (2)-(3)\\
b &= 7 &&\\
a+7 &= 16 && (1)\\
a &= 9 &&
\end{aligned}$$

This means that the complete sequence must be 9, 7, 16, 23, 39, 62.

Checking for understanding

Make up another puzzle like this one, but with the given numbers in different positions. See if your partner can use simultaneous equations to solve it. Which positions of the given numbers make the puzzle harder or easier? Why?

Students could try using more than 6 boxes or changing the 'sum of two previous numbers' rule. They could also try making a set of puzzles of gradually increasing difficulty.



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