Lesson plan: MATHS KS4

SPIDER ON A CUBOID

Understanding distances and movements in 3 dimensions is vital for living in a 3D world, says **Colin Foster**

DISCOVER

6 of the best FREE resources for 3D shape recognition at

teachwire.net/ks33d

WHY TEACH THIS?

We live in a 3-dimensional world, and yet a lot of shape and space work in maths is 2-dimensional. This lesson bridges 2D and 3D.

KEY CURRICULUM LINKS

+ use the properties of cuboids to solve problems in 3D

+ use Pythagoras' Theorem to solve problems involving right-angled triangles

In this lesson, students will investigate the shortest possible route for a spider to walk over the surface of a cuboid from one vertex to the opposite corner. They will need to draw out a net for the cuboid and use Pythagoras' Theorem to find which distance is the shortest possible. There are many opportunities to develop a logical, systematic approach and to apply their knowledge of 3D shapes.

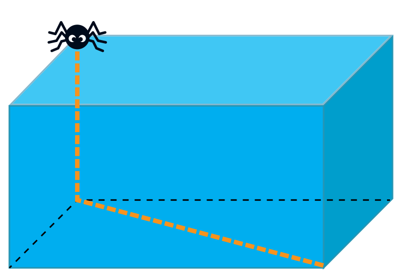
Q What is the shortest route over the surface of a cuboid from one corner to the opposite corner?

STARTER ACTIVITY

Note: If you have a student with serious arachnophobia, you might wish to change from a spider to a different (non-flying) animal!

Q Imagine a spider in the corner of the classroom (point to a corner of the ceiling in the room). Suppose it wants to get to this corner (point to the opposite corner of the room by the floor). What's the shortest way for it to get there?

If your classroom is not roughly cuboid-shaped, then you will need to adapt this scenario. The aim is for the students to describe the sort of path the spider would need to take. There is no need to do any calculations at this stage. For instance, students might suggest that the spider crawls down vertically to the floor and then diagonally across the floor, as shown in the orange path in the diagram.



MAIN ACTIVITY

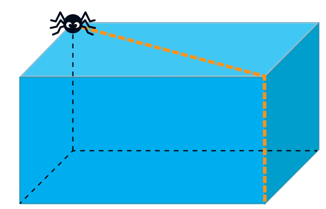
Q Let's pick some dimensions for our cuboid – say 5m by 6m on the ground by 3m high. How far is the journey in orange?

Students might find it hard to see that they need to use Pythagoras' Theorem to calculate part of the distance. They might find it hard to locate the relevant right-angled triangle or see which side is the hypotenuse. They also might forget to add on the vertical 3m at the start of the journey. They should obtain:

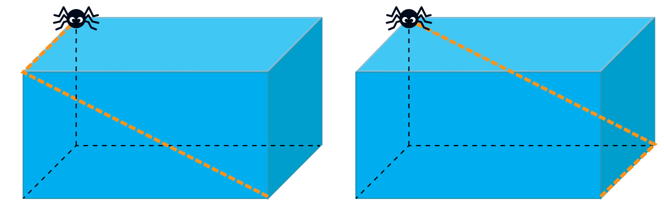
$3 + \sqrt{5^2 + 6^2} = 3 + \sqrt{61} =$ 10.8m (correct to the nearest decimal place).

Q Find some other possible routes for the spider and calculate how long they are. What is the shortest route you can find?

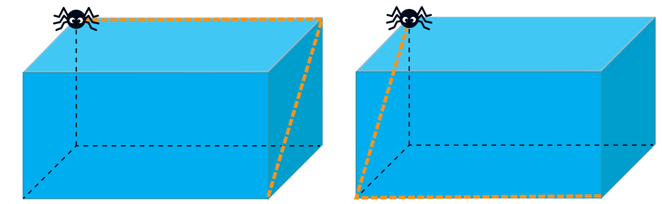
Five other possibilities, and their distances, are shown below and to the right.



This must be the same as the first route ($3 + \sqrt{61} =$ 10.8m, correct to 1 decimal place).



$5 + \sqrt{3^2 + 6^2} = 5 + 3\sqrt{5} =$ 11.7m (correct to 1 decimal place)



$6 + \sqrt{3^2 + 5^2} = 6 + \sqrt{34} =$ 11.8m (correct to 1 decimal place)

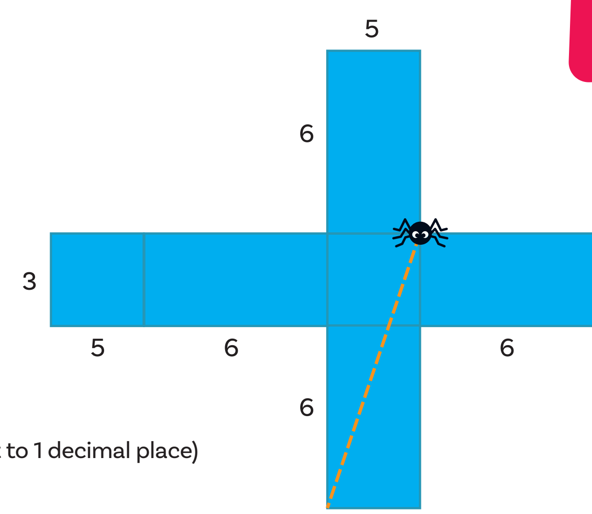
Students may be surprised that these do not all come to the same total, and, in fact, there is quite a difference between the shortest and the longest routes. Since $\sqrt{a^2 + b^2} < a + b$ (unless either a or b is zero), the total will be largest when the largest of the three lengths is **not** under the square root, so $3 + \sqrt{5^2 + 6^2} < 5 + \sqrt{3^2 + 6^2} < 6 + \sqrt{3^2 + 5^2}$.

Students who finish could choose their own dimensions for a cuboid and see if they can make the differences between the lengths of the different routes larger or smaller.

DISCUSSION

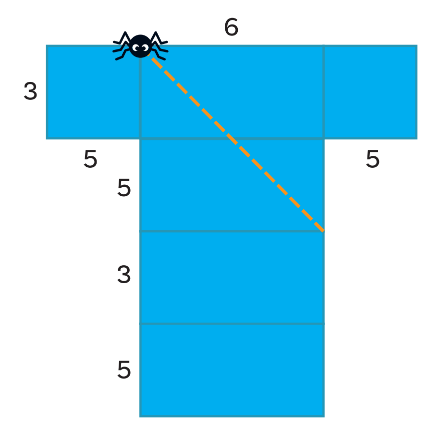
Q None of these routes is actually the shortest possible. The spider knows better than any of us! Did anyone have any routes where the spider didn't travel directly along any of the edges of the cuboid?

Opening out the cuboid into its net allows students to see a shorter route:



$\sqrt{5^2 + 9^2} =$ 10.3m (correct to 1 decimal place)

And drawing the net differently gives an even shorter one!



$\sqrt{8^2 + 6^2} =$ 10m

If this route is shortest on the net, then it will also be shortest over the surface of the cube. So, 10m is the shortest possible.

AIM

Planning for Teaching GCSE Mathematics with Mixed Attainment Groups, by Mike Ollerton and Sam Hoggard

This is a new, must-have resource for all maths departments. Don't be fooled by the title - the resources work for all groupings of students, with careful thought being given to mixed attainment classes. It encourages students to develop a deeper mathematical understanding whilst engaging them with key mathematical concepts. **www.atm.org.uk/shop/act117pk**

ADDITIONAL RESOURCES

There is a nice puzzle involving nets at **nrich.maths.org/2315.**

GOING DEEPER

Confident students could invent their own problem of this kind, perhaps over a more complicated shape than a cuboid.

ABOUT OUR EXPERT

Colin Foster is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. His website is **www.foster77.co.uk**, and on Twitter he is **@colinfoster77**.