Lesson plan: MATHS KS4

STATISTICAL PUZZLER

Understanding how different kinds of averages are calculated and behave is an important skill, says Colin Foster

In this lesson, students tackle a statistical puzzle which forces them to adjust some numbers to create certain values of the mean, mode, median and range in various small datasets. There are multiple simultaneous constraints, which makes this a challenging problem, and there are opportunities for students to create simple proofs of impossibility.

STARTER ACTIVITY

Q *Which one of these five numbers is the mean of the other four?*

1, 4, 5, 5, 5

(You may need to repeat the question for students to work out exactly what it is that you are asking.)

Students might take different groups of four numbers and calculate the means, but there is an easier way. The answer must be the mean of the ***five*** numbers, because, when the mean of a set of numbers is included into that set, it does not change the mean. So, the answer must be $\frac{1+4+5+5+5}{5} = 4$.

DOWNLOAD

a FREE KS4 lesson plan on the language of averages, at

teachwire.net/meaningmean

WHY TEACH THIS?

This lesson reviews mean, median, mode and range by offering a statistical puzzle that illuminates the connections between them.

KEY CURRICULUM LINKS

- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples
- describe, interpret and compare observed distributions of a single variable through: … appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)

Q How are mean, mode, median and range related?

MAIN ACTIVITY

Q *Let's keep the same 5 numbers as before:*

1, 4, 5, 5, 5.

*Which of these five numbers is the **mode** of the other four?*
*Which of these five numbers is the **median** of the other four?*
*Which of these five numbers is the **range** of the other four?*

Students will need to experiment with the numbers, which will generate a lot of useful practice and thinking. They may need to recap the meaning of these terms.

The answers are:

*Which one is the **mean** of the other four numbers?*	4
*Which one is the **mode** of the other four numbers?*	5
*Which one is the **median** of the other four numbers?*	5
*Which one is the **range** of the other four numbers?*	1 or 4

Q. *Now, can you make up a set of 5 numbers that gives **one different answer** to each of these questions? So, there should be just one answer to each question, and each answer should be different from all the others.*

In the example given, 5 is the answer to two of the four questions, and the fourth question has two answers (1 and 4). The challenge is to find a set of numbers so that each of these questions has just one answer, and all the answers are different.

Give students time to experiment and encourage them to check the calculations on each other's sets of numbers.

It turns out to be impossible, but the challenge is to explain why. Since the mode of four of the numbers must be equal to the fifth number, this means that at least ***three*** of the numbers must be equal, not just two. And this means that, writing the numbers in order, we must have one of these three possibilities:

1. mode = mode = mode ≤ $high_1$ ≤ $high_2$
2. low ≤ mode = mode = mode ≤ high
3. low_1 ≤ low_2 ≤ mode = mode = mode

And, in each of these cases, if you remove any of the 'low' or 'high' numbers, the median of the remaining four numbers has to be the mode, because in each case you have

something ≤ mode ≤ mode ≤ something

So, you will never be able to find a number ***not*** equal to the mode which can be the median of the other four numbers. That's a shame!

This argument doesn't involve any intricate algebra, but it is quite sophisticated reasoning, and hard to think through, and students will need support in developing this kind of argument.

Q. *What if you are allowed to have more than 5 numbers?*

Give students plenty of time to explore and see what they can come up with. Encourage them to write down clearly how many numbers they are using at any point, and what they have found out. If they think that something is impossible, they should try to explain why, and not just give up!

DISCUSSION

Q *What did you find out? Which sets of numbers worked and which didn't? Why?*

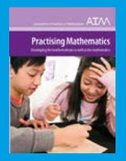
With six numbers, you have the same problem as with five, that there must be at least three equal numbers, so as to give a mode when one of them is removed, and this means that, when you remove a different number, the median of the five remaining numbers must also be equal to the mode, because the middle number must be one of the repeated numbers.

With more numbers, it does eventually become possible. For example, with 10 numbers, you could have something like

1, 1, 1, 1, 2, 2, 3, 4, 7, 8

*Which one is the **mean** of the other nine numbers?*	3
*Which one is the **mode** of the other nine numbers?*	1
*Which one is the **median** of the other nine numbers?*	2
*Which one is the **range** of the other nine numbers?*	7

Here, each question has only one answer, and all of the answers are different.



Practising Mathematics – Developing the mathematician as well as the mathematics (by Dave Hewitt and Tom Francome)

This book from the Association of Teachers of Mathematics is full of ideas for practising the content of the secondary maths curriculum. It contains tasks and activities for number work, ratio and proportion, algebra, geometry, probability and statistics and includes 100+ downloadable supporting whiteboard slides.
atm.org.uk/shop/act107pk

ADDITIONAL RESOURCE

There is a very nice mean-mode-median-range Venn diagram task at: mathsvenns.com/mean-median-mode-range

GOING DEEPER

Confident students could try to categorise what is possible and impossible for different total numbers of numbers.

ABOUT OUR EXPERT

Colin Foster is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. His website is www.foster77.co.uk, and on Twitter he is @colinfoster77.