Lesson plan: Mathematics KS4

SURVIVING IN THE DESERT

Working out how to carry enough food to stay alive in a challenging environment poses interesting problems

Check your Y7 learners' understanding of ratio with a mental arithmetic exercise: **teachwire.net/ks2ratio**

WHY TEACH THIS?

Mathematical modelling using ratio can be a powerful way for students to see the application of mathematics to the real world.

KEY CURRICULUM LINKS

+ Model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions

In this lesson, students use their knowledge of ratio to make sense of a real-world scenario in which enough food must be carried into the desert to allow a group of people to survive for a certain length of time.

STARTER ACTIVITY

Q *Look at this puzzle:*

If it takes 4 people 3 days to dig a hole 4 metres by 6 metres by 1 metre, how long will it take 2 people to dig a hole 2 metres by 2 metres by 2 metres?

Students will probably think that this seems extremely complicated.

Q *Try to work this out in pairs. Write down anything you need to* ***assume****.*

Students may realise that the volume of the second hole is 1/3 of the volume of the first hole, meaning that it would take the same 4 people only 1 day to dig it. However, there are only 2 people, so it will take them twice as long as that; in other words, 2 days.

To solve this idealised problem, students have to assume that everyone digs equally hard, the ground is uniformly solid, the holes are cuboids, and many other things that they will be able to think of.

Q How long can you survive in the desert if you have to carry all your food?

MAIN ACTIVITY

Display this quote from the autobiography of the naturalist and broadcaster David Attenborough:

> If one person who is not carrying food is accompanied by two others carrying full loads of provisions, the three of them will have enough food to last a fortnight. (Attenborough, 2010, p. 223)

Q What can you work out from this? What do you need to assume?

Here are some questions that students might think of:

- How long could they last if all three of them carried food?
- How long could one person last, all on their own?
- How many people would you need to take with you if you wanted to last 3 weeks?
- How many people would you need if you wanted to last 5 weeks?

This is quite difficult. Attenborough describes having 2 people carrying, but 3 people eating, and says that they can last all together for 14 days. Let's assume that everyone eats exactly the same amount of food, regardless of whether they are carrying things or not. In that case, if they all carried food, they would eat exactly the same amount as before, but they would have half as much food again, so they would be able to last half as long again; i.e., 14 × 1.5 = 21 days. Since now everyone could just eat the food that they personally have carried, this means that an individual person could also last by themselves for 21 days.

Students might calculate how long the group can survive with different numbers of carriers and eaters – some values are shown in the table below.

	Number of eaters									
Number of carriers	1	2	3	4	5	6	7	8	9	10
1	21	10.5	7	5.25	4.2	3.5	3	2.625	2.33	2.1
2		21	14	10.5	8.4	7	6	5.25	4.67	4.2
3			21	15.75	12.6	10.5	9	7.875	7	6.3
4				21	16.8	14	12	10.5	9.33	8.4
5					21	17.5	15	13.125	11.67	10.5
6						21	18	15.75	14	12.6
7							21	18.375	16.33	14.7
8								21	19.67	16.8
9									21	18.9
10										21

Number of days (rounded, where necessary) that it is possible to survive with different numbers of carriers and eaters

A formula to capture what is going on is

$$\text{time (days)} = \frac{21 \times \text{number of carriers}}{\text{number of eaters}}$$

We can see from this that it is impossible to survive for more than 21 days, no matter how many people you take with you, because the number of carriers cannot exceed the number of eaters (everyone has to eat, even if they don't all carry!).

DISCUSSION

You could conclude the lesson by discussing how students made sense of the information that they were given and how they reasoned about the problem.

Q *What did you do first? What did you find difficult? What helped you to make progress with the problem? What have you learned that could help you to solve similar problems in the future?*

You could ask students to devise a problem that is similar in its mathematical structure to this one, even though the context and details could be completely different. This is a difficult task, but a good one to encourage students to attend to deeper mathematical structure.

GOING DEEPER

Confident students could consider the case where there is always one more eater than carrier. If *n* is the number of carriers, then students could plot a graph of survival time against *n*. They will discover a ***horizontal asymptote*** at 21 days.

ADDITIONAL RESOURCES

For more details on the survival problem, see Foster (2017). Another interesting and quite open problem relating to crossing a desert is the so-called *'jeep problem'* – see **mathsisfun.com/puzzles/cars-across-the-desert.html**

REFERENCES

Attenborough, D. (2010) *Life on Air: Memoirs of a broadcaster.* Revised and Updated Edition. Bungay: BBC Books.

Foster, C. (2017). Carrying your provisions. *Mathematics in School, 46*(1), 30.

THE AUTHOR

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