[ MATHS PROBLEM ]

# SYMMETRY

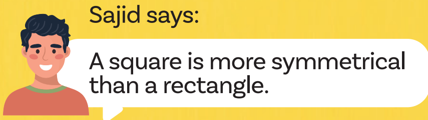
In everyday life, things are often described as either 'symmetrical' or not. But in mathematics it's a bit more complicated than that. This lesson aims to help students to sort out confusions with line symmetry and rotational symmetry.

**Imprecise use of language, such as saying that 'a hexagon is symmetrical', may be tricky to unpick. This lesson focuses students on being precise about what exactly they mean when they use the word 'symmetry' in mathematics.**

## THE DIFFICULTY

This task is intended to bring to the surface the need for precision when talking about the symmetry of a shape.

Sajid says:

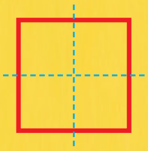



*What might he mean by this?*
*Do you agree with his statement?*
*Why / why not?*

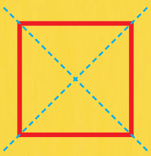
This is a deliberately ambiguous and imprecise statement. To begin with, *all* squares are rectangles, since a rectangle is any 4-sided polygon with 4 right angles, and that includes all squares. Students will probably agree that all squares, regardless of size or orientation, are 'equally' symmetrical. But how does the symmetry of a *non-square rectangle* (sometimes called an ***oblong***) compare with that of a square?

## THE SOLUTION

Students will agree that a square has **4 lines of reflection symmetry**:

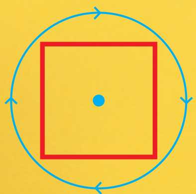


two perpendicular lines of symmetry passing through the centre of the square and *parallel to pairs of opposite sides*



two perpendicular lines of symmetry passing through the centre of the square and *passing through opposite vertices*

and **order 4 rotational symmetry** about the centre of the square:



This may be a good opportunity to encourage accurate use of terms such as *side, vertex, parallel, perpendicular, right angle, rotate, (anti)clockwise and centre.*

An oblong, even though it still has 4 sides and 4 vertices, has only **2** lines of symmetry and ***order 2*** rotational symmetry. Students might disagree initially, and want to include the diagonals of the rectangle. Taking a piece of A4 paper and folding along the diagonal should convince them that the diagonals are ***not*** lines of symmetry.

The best way for students to develop their sense of line and rotational symmetry is for them to be challenged to find or invent shapes with various different possible symmetries. This forces them to consider what is possible and what isn't. Students could devise their own questions, but here are two to begin with:.

### 1. Is the number of lines of symmetry always equal to the order of rotational symmetry?

Students will find that this is true for the regular polygons (like a square) – a regular *n*-gon has *n* lines of symmetry and order *n* rotational symmetry – and also for some ***irregular*** shapes, such as a rectangle (2 and 2) and any completely unsymmetrical shapes (1 and 1). It is possible to find shapes with ***any*** order of rotational symmetry but ***no*** lines of symmetry, such as

which has order 4 rotational symmetry, but no lines of symmetry. However, the reverse (e.g., 4 lines of symmetry but no rotational symmetry) is ***not*** possible, and students should be challenged to think about why.

### 2. When there are exactly 2 lines of symmetry, are they always perpendicular to each other?

This turns out to be true, and students could explore what happens with more than 2 lines of symmetry.

**Checking for understanding**

These questions should enable students to demonstrate what they have learned from thinking about this:

*I'm thinking of a shape. It has 3 lines of symmetry. What orders of rotational symmetry might it have?*

*Now I'm thinking of another shape. It has order 5 rotational symmetry. How many lines of symmetry might it have?*

Colin Foster (@colinfoster77) is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. foster77.co.uk