WHY TEACH THIS?

It is common for students to approach many mathematical puzzles by trial and improvement, and that can be a very good way to get a sense of what the problem is about and what the possibilities might be. But students will see in this lesson that algebra can often provide a more powerful and efficient approach.

THE POWER OF PUZZLES

ALGEBRA IS OFTEN HATED AND FEARED BY STUDENTS, BUT THIS LESSON ALLOWS THEM TO SEE HOW IT'S A SKILL THAT CAN SOMETIMES MAKE LIFE EASIER RATHER THAN HARDER, SAYS COLIN FOSTER...

Sometimes students think that algebra exists solely to make their lives a misery! They are often presented with questions like "Simplify $x + x + y$", which may seem to have nothing to do with the world as they know it. The correct answer $2x + y$ may not look any more correct than an incorrect answer like $2xy$. (In fact, $2x + y$ may look a bit incomplete, as there is still a plus sign left.) And the correct answer doesn't seem to help in any way or contribute to anything more than getting a tick on a test! In this lesson, students are given some numerical puzzles that may not initially appear to have anything to do with algebra. Students may solve them however they like. Some may be able to do intuitively, by intelligent trial and error, but where the structure is more complicated, or the answers are not easy integers, they will struggle to solve them with their native wit. This is where a diagram or some symbols may really help. The key idea is to represent what you don't yet know by a symbol (not necessarily an "x" – it could be a picture) and then proceed with the symbol, rather than with a specific number. Very quickly this leads to a solution – algebra to the rescue!

+KEY RESOURCE

- **ATM has developed a collection of over 50 (photocopiable) algebra games that support the development of algebraic thinking. The activities are ideal for whole class and collaborative group work. By playing the games, learners can be introduced to new and challenging algebra concepts in ways that are both enjoyable and thought provoking. www.atm.org.uk/shop/act055**
- **For another fantastic resource look at Grid Algebra Software, which is a unique piece of software to support and develop understanding of early algebra. www.atm.org.uk/shop/sof074**
- **ATM members can also access the free CPD support packs which include ideas for a staff workshop on algebra. www.atm.org.uk/CPD-Support-Packs**

STARTER ACTIVITY

Pen and ruler

Q. *Here is a puzzle for you to try:*

I bought a pen and a ruler.
The pen cost £1 more than the ruler.
The total cost was £1.10.
How much did the ruler cost?

Some students will quickly say that the answer is 10 pence, but this is wrong. You could handle this by asking them to think a bit more, or by asking what other people think.

Q. *Does anyone think the answer* ***isn't*** *10 pence? Why?*

If everyone agrees that the answer is 10 pence, you could ask:

Q. *If the ruler costs 10 pence, how much does the pen cost?*

If the ruler costs 10p, then the pen costs £1 more, which is £1.10. So altogether the two items would cost £1.20. So 10p ***can't*** be the right answer. Students could think about it some more in pairs. They may realise that the answer must be ***less than*** 10p.

The ruler must have cost 5p, so that the pen would cost £1.05. Then the total cost is £1.10.

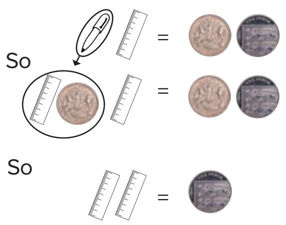
Students might regard this as a "trick" question, and if so you could ask them why.

Q. *How did you work it out? How would you do it if the numbers were different?*

Students might have used trial and improvement. Some might have thought about it symbolically, perhaps using objects from their pockets and pencil cases:

So

So



So a ruler must cost 5 pence.

They might even have used formal algebra, with r representing the cost of the ruler in pounds, say: $r + (r + 1) = 1.1$ means that $2r = 0.1$, so $r = 0.05$. So the ruler costs 5 pence.

Can students still solve the problem if we change £1.10 to £2.50 or something else? (What if we change the £1 as well?) Is there a general method? If students can't do this yet, leave it unresolved, as this will be addressed in the main activity.

MAIN ACTIVITY

Give students these four puzzles:

I'm thinking of two numbers. One number is 8 more than the other number. The sum of the two numbers is 32. What are the two numbers?	I'm thinking of two numbers. One number is 7.3 more than the other number. The sum of the two numbers is 30. What are the two numbers?
I'm thinking of two numbers. One number is 3.5 times the other number. The sum of the two numbers is 36. What are the two numbers?	I'm thinking of three numbers. The second number is 2 more than the first number. The third number is 3 times the second number. The sum of the three numbers is 54. What are the three numbers?

A task sheet containing these is available at teachsecondary.com/downloads/maths-resources

Q. *Choose any one of these puzzles. Try to solve it. See if you can solve it in more than one way. Then try the others.*

Students will need calculators in order to experiment freely and locate solutions. They will need to know that "sum" means the total. They might query whether "8 more" means "8 times more". If they do, you could ask them "What is 8 more than 10?" They will say 18, not 80, so that should resolve their query. If they get stuck on one puzzle, they could try a different one and come back to it.

They may be able to solve the first one quite quickly by trial and improvement, but the others will be more challenging. If they are stuck, you could ask whether they could draw a diagram to help them or use some objects from their pencil cases to represent the unknown numbers.

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about what they have learned. Did they find any methods other than trial and improvement?

The answers to the four puzzles are:

12 and 20	11.35 and 18.65
8 and 28	9.2, 11.2 and 33.6

Students could represent the ***smallest*** **number by a symbol, or a letter such as x, so that subtraction and division are not needed to represent the other number(s). They might write and solve equations such as $x + (x + 2) + 3(x + 2) = 54$, or they might make drawings where they represent the unknowns symbolically, as shown with the pen and ruler above.**

An algebraic solution may be quicker and easier. It also has the advantage of showing you that you have found the ***only*** **solution, whereas with trial and improvement it may be hard to be sure that you haven't overlooked other possible answers.**

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is an Assistant Professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

STRETCH THEM FURTHER

IF STUDENTS SOLVE ALL FOUR PUZZLES, ASK THEM:
Q. *CAN YOU MAKE UP AN* ***EASIER*** *PUZZLE LIKE EACH ONE?*
Q. *CAN YOU MAKE UP A* ***HARDER*** *PUZZLE LIKE EACH ONE?*

THEN THEY CAN SEE IF THEY CAN SOLVE EACH OTHER'S PUZZLES. THEY COULD HAVE TWO MYSTERY NUMBERS, OR MORE THAN TWO, IF THEY LIKE. THESE CAN BE QUITE EASY TO MAKE UP BY STARTING WITH THE NUMBERS BUT MUCH HARDER FOR THE OTHER PERSON TO SOLVE!

ADDITIONAL RESOURCES

A NICE VISUAL WAY OF THINKING ABOUT SOLVING EQUATIONS IS AVAILABLE AT HTTP://NLVM.USU.EDU/EN/NAV/FRAMES_ASID_201_G_3_T_2.HTML A TASK SHEET CONTAINING THE FOUR PUZZLES IS AVAILABLE AT TEACHSECONDARY.COM/DOWNLOADS/MATHS-RESOURCES

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