[MATHS PROBLEM]

THE QUADRATIC FORMULA

Students often make errors when using the quadratic formula

In this lesson, students learn to make sense of the quadratic formula by working backwards

THE DIFFICULTY

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to solve this equation: $5x^2 - 3x - 2 = 0$.
Now check your answer by substituting back into the original equation.

It is very easy to make mistakes, particularly with the negative signs, and not obtain $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\times5\times(-2)}}{2\times5}$, which gives $x = 1$ or $x = -\frac{2}{5}$. Many incorrect answers are possible by failing to calculate $-b = 3$ or $b^2 = 9$ for instance.

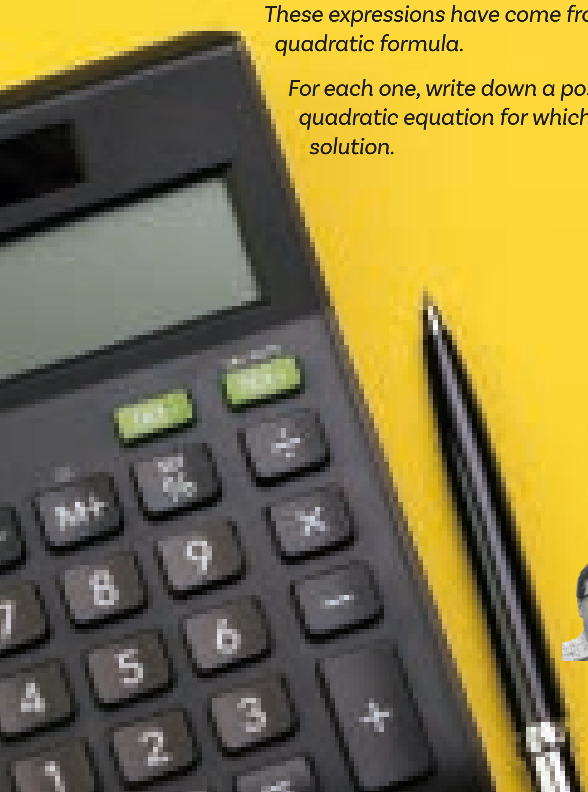
If students find this too easy, let them try $-5x^2 + 3 - 2x = 0$. The solutions are: $x = -1$ or $x = \frac{3}{5}$

Here, the terms do not appear in the standard form of $ax^2 + bx + c = 0$, so students may incorrectly take $b = 3$ and $c = -2$, rather than $b = -2$ and $c = 3$. In addition to this, a negative coefficient of x^2 may also be confusing, and one option here is to multiply both sides of the equation by -1 to obtain $5x^2 + 2x - 3 = 0$, with $a = 5$, $b = 2$ and $c = -3$.

THE SOLUTION

These expressions have come from using the quadratic formula.

For each one, write down a possible ***simplified*** *quadratic equation for which they are the solution.*



$\frac{-(-3) \pm \sqrt{(-3)^2 - 4\times2\times1}}{2\times2}$	$\frac{-2 \pm \sqrt{4 + 4\times3\times1}}{2\times3}$
$\frac{3 \pm \sqrt{(-3)^2 - 4\times2}}{2\times1}$	$\frac{-3 \pm \sqrt{9 - 4\times2}}{4}$
$\frac{2 \pm \sqrt{4 - 4\times(-3)}}{2}$	$\frac{-1 \pm \sqrt{1 - 4\times3\times(-2)}}{6}$
$\frac{1 \pm \sqrt{1 - (-24)}}{-4}$	$\frac{-2 \pm \sqrt{16}}{-6}$

A sheet containing these expressions is available at XXXX.

These expressions gradually get harder, as they become increasingly simplified and students need to look harder to find a, b and c within the expressions.

The answers are:

$2x^2 - 3x + 1 = 0$	$3x^2 + 2x - 1 = 0$
$x^2 - 3x + 2 = 0$	$2x^2 + 3x + 1 = 0$
$x^2 - 2x + 3 = 0$	$3x^2 + x - 2 = 0$
$-2x^2 - x + 3 = 0$	$-3x^2 + 2x + 1 = 0$

Any constant multiple of these equations would also be correct.

This task forces students to unpick the method by 'thinking backwards' and noticing small differences in how the signs appear in each of the solutions.

Checking for understanding
Solve this equation using the quadratic formula: $x^2 - x - 1 = 0$

Students should obtain the solution $x = \frac{1 \pm \sqrt{5}}{2}$.

They should now be more confident in correctly using the quadratic formula, and less easily confused by negative quantities.

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