THE SIMPLE LIFE

WHY TEACH THIS?

Exercises are not the only way to improve students' fluency in mathematical procedures – sometimes the very same skills can be developed in a more exploratory fashion.

GAINING ESSENTIAL SKILLS IN ALGEBRAIC MANIPULATION DOESN'T HAVE TO ENTAIL TEDIOUS DRILL AND PRACTICE, SAYS COLIN FOSTER

Expanding brackets and collecting like terms to simplify expressions are perhaps not the most exciting of mathematical topics, but fluency in these skills is needed in order to solve equations and engage with more stimulating mathematical problem solving. Unless students take the time to master these skills they will be disadvantaged later. But how can we avoid lessons on algebraic manipulation descending into endless mindless drill and practice? One way is to engage students in *devising* expressions that will simplify to produce a given result – in this lesson the expression "5x + 8y". Restricting the possible expressions that can be combined to make this to five given linear binomials forces students to engage in some careful trial and improvement. To find all the possible solutions, they will have to engage with negative coefficients, leading to plenty of opportunities for strengthening their skills in expanding and simplifying algebraic expressions.

STARTER ACTIVITY

Q. I would like you to expand and simplify these four expressions. Which one is the odd one out? If you finish, try to make up some more that fit the pattern.

(a) $(3x + 4y) + 2(x + 2y)$
(b) $4(2x + 5y) – 3(x + 4y)$
(c) $3(2x + 3y) – (x – y)$
(d) $3(x + 3y) + (2x – y)$

A task sheet containing all of the problems in this lesson is available at teachwire.net/thesimplelife

This starter could work well if students have recently been working on this topic. If they aren't too familiar with this, you might first need to ask if one student can remind everyone else what 'expand' and 'simplify' mean. A student could be asked to make up a similar but different example and come to the board and show everyone how to work it out.

Students could try the task individually or in pairs. When they multiply out the brackets and collect like terms they should find that (a), (b) and (d) all come to $5x + 8y$. The odd one out is (c), which comes to $5x + 10y$. Students might get $5x + 8y$ for this one as well, if they fail to realise that the final y must be ***added***, because we are ***subtracting a negative*** y.

Q. Was anything tricky about these? Did you make any mistakes – or ***nearly*** *make any mistakes – when you were doing them? What things do you have to be careful of when simplifying expressions like this? Do you have any advice for someone doing questions like this?*

This is an opportunity to gauge students' confidence and facility with this skill. It's OK if not everyone is very proficient yet – this is the procedure that they will get better at during today's lesson. On the other hand, if they are confident with this skill then they will be able to focus more of their attention on the problem-solving aspects.

MAIN ACTIVITY

Q. The answers to all of the questions today are going to be "$5x + 8y$". Your job is to make up the questions! The only brackets that you are allowed to use are:

$(x + y)$ $(x + 2y)$ $(x - 2y)$
$(x + 4y)$ and $(2x + 3y)$

(The colours are not essential – they are just to help you see which bracket is which.)

You can pick any two of these brackets and then put numbers in front of them, and a plus or minus, to make an expression. For example, you could choose the brackets $(x + 2y)$ and $(x + 4y)$:

❑ $(x + 2y) \pm$ ❑ $(x + 4y) =$

Now we need to find numbers to go in the boxes.

Ask students to suggest some numbers and use whatever numbers they offer, provided that they are reasonably small integers. Ask the students to decide whether the numbers they give you are positive or negative and then work through the expanding and simplifying process together on the board. It is very unlikely that the answer will come to $5x + 8y$.

Q. Can you choose a different pair of numbers so that we get an answer of $5x + 8y$?

This is quite hard!
Students might be able to get either the coefficient of x correct or the coefficient of y correct but not both at the same time!

The correct answer is to choose 6 and –1 as the numbers to go in the boxes, so that you have
$6(x + 2y) - (x + 4y) = 5x + 8y$.

If no one can immediately do it, this might be a good point to ask the class to work on the problem in pairs. Students will need time to experiment, and in the process (even when they are not even getting close to $5x + 8y$) they will be getting a lot of practice at expanding and simplifying, which is the main purpose.

If they succeed, then they can try choosing different pairs of brackets. They are aiming to find a way to combine them to make $5x + 8y$ every time.

There is a lot for students to try here, and confident students can try using three brackets, four brackets or even all five!

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is an assistant professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

ADITIONAL RESOURCES

A TASK SHEET CONTAINING ALL OF THE PROBLEMS IN THIS LESSON IS AVAILABLE AT TEACHWIRE.NET/THESIMPLELIFE

STRETCH THEM FURTHER

Making up a problem like these, with an answer different from $5x + 8y$, can be a good challenge.

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about their answers and how they got them.

*Q. Who found a way of making 5x + 8y? Did anyone find more than one way? Come and show us how you did it? Which brackets did you use? Which numbers did you use? How did you combine them? Why did you choose those numbers? Did anyone get **close** to 5x + 8y? Show us what you did.*

There are 7 solutions in which both numbers are integers:

$$4(x + y) + (x + 4y) = 5x + 8y$$
$$2(x + y) + 3(x + 2y) = 5x + 8y$$
$$(x + 2y) + 2(2x + 3y) = 5x + 8y$$
$$3(2x + 3y) - (x + y) = 5x + 8y$$
$$2(x - 2y) + 3(x + 4y) = 5x + 8y$$
$$6(x + y) - (x - 2y) = 5x + 8y$$
$$6(x + 2y) - (x + 4y) = 5x + 8y$$

There are also 3 more solutions in which both numbers are ***not*** integers. These are much harder to find, but pupils confident with simultaneous equations might be able to obtain them:

$$\tfrac{9}{2}(x + 2y) + \tfrac{1}{2}(x - 2y) = 5x + 8y$$
$$\tfrac{18}{7}(2x + 3y) - \tfrac{1}{7}(x - 2y) = 5x + 8y$$
$$\tfrac{1}{5}(x + 4y) + \tfrac{12}{5}(2x + 3y) = 5x + 8y$$

For example, for the first one they could write

$$a(x + 2y) + b(x - 2y) = 5x + 8y.$$

By comparing coefficients of x and y, this gives $a + b = 5$ and $2a - 2b = 8$, so we have

$$a + b = 5, \text{ and}$$
$$a - b = 4.$$

Solving these simultaneously gives $a = \tfrac{9}{2}$ and $b = \tfrac{1}{2}$.

Students might also be able to find solutions involving more than two different brackets; for example:

$$(x + y) + 2(x + 2y) + (2x + 3y) = 5x + 8y$$

or maybe even one using all five brackets, such as:

$$2(x + y) + 4(x + 2y) - 2(x - 2y) - 3(x + 4y) + 2(2x + 3y) = 5x + 8y.$$