[ MATHS PROBLEM ]

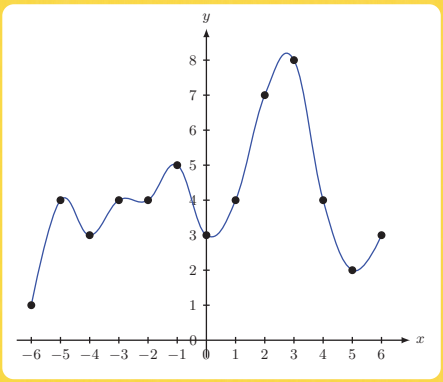
# TRANSFORMING GRAPHS

When transforming graphs, the graph often moves in the opposite direction to what students expect, says **Colin Foster**

**In this lesson, students understand why the graph of $y = f(x)$ moves to the left when it is transformed into $y = f(x + a)$, where $a$ is a positive constant**

## THE DIFFICULTY

*Look at this graph of a function $y = f(x)$.*




*What would the graph of $y = f(x + 1)$ look like? Why?*

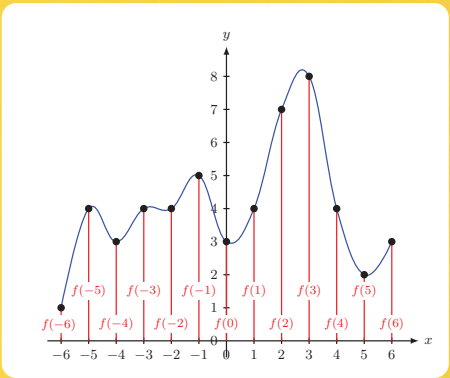
Students are often confused by this, and think the graph would translate 1 unit to the right. Actually, it will translate 1 unit to the left.

## THE SOLUTION

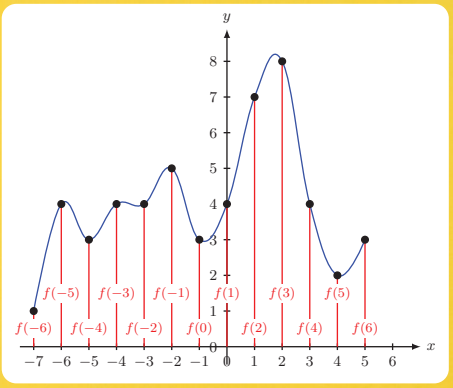
*Look at the red vertical lines in this graph.*

*What do they represent?*

The red lines are the $y$ values, which are the $f(x)$ values, for particular (integer) values of $x$. For example, $f(3)$ is the $y$ value when 3 is substituted into the function $f$.




Let's imagine translating the graph, and all its $y$ values, 1 unit to the left, as shown below.




Now, $f(3)$ no longer appears at $x = 3$; it appears at $x = 2$. So, this graph must now be $y = f(x + 1)$. Each $x$ value steals the $f(x)$ value from the $x$ value that is 1 higher, so the $y$ value becomes $f(x + 1)$ instead of $f(x)$. Translating to the left means stealing $y$ values associated with higher $x$ values.

### Checking for understanding

Make a similar argument for what will happen if we transform $y = f(x)$ into $y = f(2x)$.

Here, $f(6)$ will be dragged in towards the $y$ axis until it reaches $x = 3$. All the red lines will move inwards until their separation is $\frac{1}{2}$, rather than 1. So, the transformation will be a stretch with scale factor $\frac{1}{2}$ parallel to the $x$ axis.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