Lesson plan: MATHS KS3

TRAPEZIA ACTS

Students need to understand the reasons behind the formulae for the area of different quadrilaterals, says **Colin Foster**

DOWNLOAD
a FREE Powerpoint for KS3 maths on properties of quadrilaterals, at
teachwire.net/ks3propquads

WHY TEACH THIS?

Understanding area is critical to doing geometry

KEY CURRICULUM LINKS

+ derive and apply formulae to calculate and solve problems involving area of trapezia

In this lesson, students will explore ways of calculating the areas of different quadrilaterals, beginning with parallelograms as the key shape, leading to triangles (as half a parallelogram) and squares and rectangles (as special parallelograms). Finally, they will look at trapezia, which, like parallelograms, can also be thought of as formed from two triangles, but in this case the triangles are not congruent. The aim of the lesson is to give students a sound feeling for area, as opposed to simply memorising formulae.

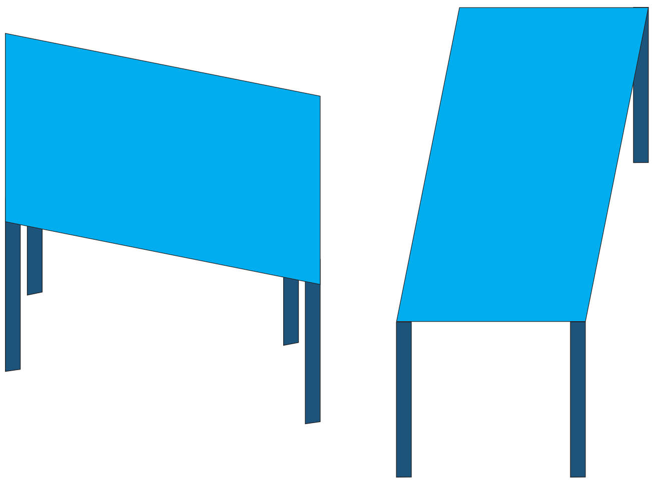
Q How can we find the area of different quadrilaterals?

STARTER ACTIVITY

Q Can you describe these two tables?

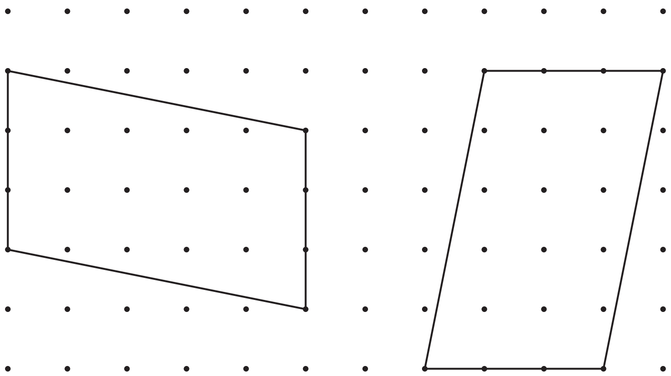
These drawings are available at teachwire.net/ks3trapezia to display on the board or hand out on paper.

This is known as the *Shepard's table illusion*. Students will probably think that the table on the left is short and fat, whereas the table on the right is long and thin. However, ***on the page***, the two parallelograms are exactly the same shape and size. This is a trick of perspective – students might find this hard to believe and may need you to verify it by dragging one on top of the other!



MAIN ACTIVITY

Q We can't always trust our eyes to tell us how big a shape is. But we can *reason* whether two shapes are the same area or not. Are these two shapes the same area or different areas? Why?



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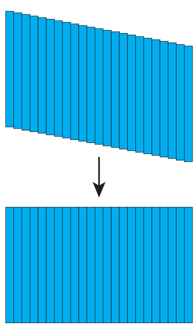
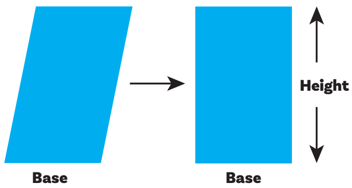
Give students squared or square-dotty paper and ask them to think about this in pairs. If they think that the shapes are the same area, can they draw some more with the same area as these? If they think the two shapes have different areas, they should decide which is bigger and by how much. (These are the same parallelograms as used in the table illusion, so they do have the same area.)

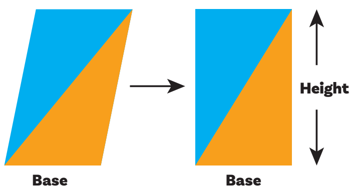
Students may reason that the areas are the same because the right-hand shape is a 90° rotation of the left-hand shape, and rotation preserves area. Or they might imagine shearing the parallelograms into 5 × 3 rectangles, since shearing preserves area. If students doubt this, a drawing like this might convince them:



If students see how a parallelogram can always be sheared into a rectangle, then it should be clear that its area is the same as the area of the rectangle it gets sheared into. This helps with seeing the base and the height (not the side length) of the original parallelogram that need to be multiplied:



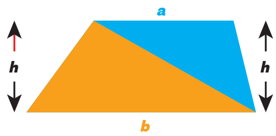
Once parallelograms are understood, rectangles and squares are sorted (because they are both special cases of rectangles) and triangles are also easy, because every triangle is half of a parallelogram:



So, if the area of a parallelogram (including rectangles and squares) is bh then the area of a triangle must be $\frac{1}{2}bh$.

This leaves trapezia, which, like parallelograms, are also made up of two triangles, but in this case the two triangles are not necessarily congruent. (If they are, the trapezium is a parallelogram.)

If you think of a general (non-isosceles trapezium) in this way, then it is clear that the area is the sum of the areas of the two triangles:



Blue area = ½ ah
Orange area = ½ bh
Total area = ½ $(a+b)\,h$

I find that this approach is much easier for students than dissection proofs where pieces have to be cut off and rearranged to make rectangles or parallelograms.

Q On squared paper, draw 10 different trapezia that all have an area of exactly 24 cm².

This is a good challenge, especially if students see parallelograms, rectangles and squares (but not triangles!) all as special cases of trapezia, and therefore legitimate responses to this task.

DISCUSSION

Q *What different trapezia did you think of? Did anyone think of a parallelogram or a rectangle or a square? Did anyone use an isosceles trapezium? Or a right-angled trapezium? How do you know that the areas of your trapezia are all definitely 24 cm²?*

The intention is that the insights from this lesson mean that students will be thinking more about **why** the areas are what they are, in terms of shearing and composition from triangles, rather than mindlessly substituting numbers into formulae.

ADDITIONAL RESOURCES

Keen students could look at **bit.ly/3041de3** to find out more about the Shepard's table illusion.

Teachers might be interested in the article: Foster, C. (2004). Trapezium artist: Some thoughts on the formula for the area of a trapezium. Mathematics in School, 33(5), 6–7, which is available free at **bit.ly/2Y6cvwm**.

GOING DEEPER

Confident students could derive a method for finding the area of a kite.

ABOUT OUR EXPERT

Colin Foster is a Reader in Mathematics Education at the Mathematics Education Centre at Loughborough University. He has written many books and articles for mathematics teachers. His website is **www.foster77.co.uk**, and on Twitter he is **@colinfoster77**.