

[MATHS PROBLEM]

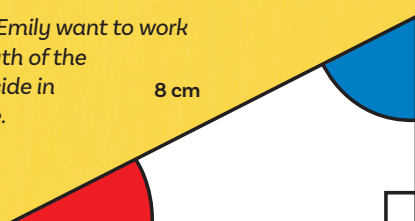
TRIGONOMETRY IN RIGHT-ANGLED TRIANGLES

Students often muddle up the different sides when doing trigonometry in right-angled triangles, says **Colin Foster**

In this lesson, students explore the relationships between the acute angles and the relevant sides in right-angled triangles

THE DIFFICULTY

Shaun and Emily want to work out the length of the **horizontal** side in this triangle.



What kind of triangle is it?

Right-angled.

Which side is the horizontal side?

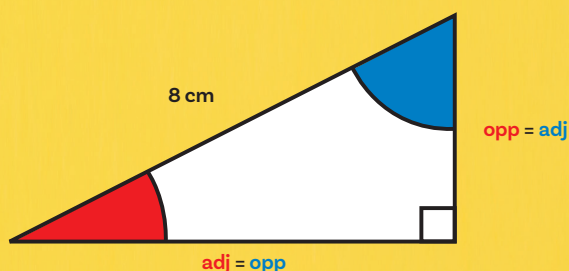
The one at the bottom.

Shaun says he needs to know the size of the **red** angle to work it out. Emily says she needs to know the size of the **blue** angle to work it out. Who is right?

In fact, either angle can be used, depending on whether they use **cosine** (red angle) or **sine** (blue angle).

THE SOLUTION

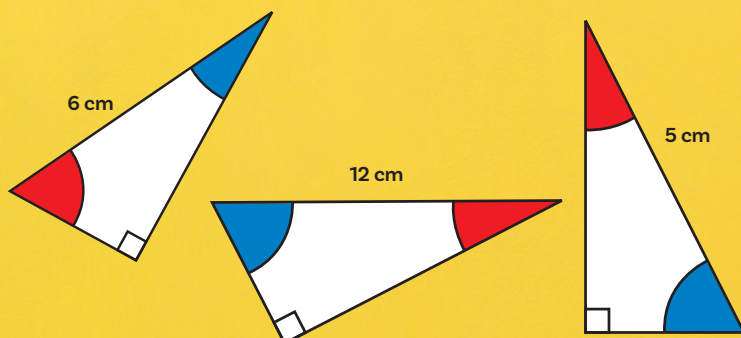
What are the coloured labels on this diagram showing?



These labels are showing that:

- the **horizontal** side is **adjacent** to the **red** angle and **opposite** to the **blue** angle.
- the **vertical** side is **adjacent** to the **blue** angle and **opposite** to the **red** angle.

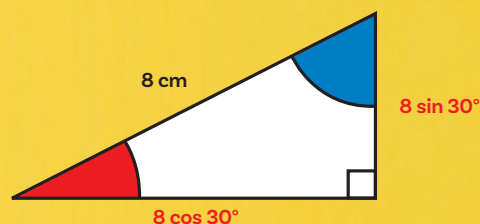
Write **adj** and **adj** and **opp** and **opp** on the sides of these triangles:



Estimate possible values for the sizes of the **red** and **blue** angles in Shaun and Emily's triangles.

Go with whatever sensible sizes are suggested (e.g. **red** = 30°, **blue** = 60°).

Here is what Shaun wrote, using the **red** angle:



What did Emily write, using the **blue** angle?

Emily wrote **8 sin 60°** on the horizontal side and **8 cos 60°** on the vertical side.

This helps students to see that, in general, $\sin(90^\circ - x) = \cos x$ and $\cos(90^\circ - x) = \sin x$, so there are always two ways to find each side.

Do the same thing for the three other triangles given above.

Checking for understanding

A right-angled triangle has a hypotenuse of 10 cm and one of its angles 35°.

Find the lengths of **both** of the other sides in **two different ways** for each.

The answers are 5.74 cm and 8.19 cm (correct to 2 decimal places).



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