[MATHS PROBLEM]

UNDERSTANDING INDICES

Students often misapply the 'rules of indices' – for example, multiplying indices when they should be added. This lesson aims to help students rely less on rules and more on understanding.

When a student sees the multiplication symbol in an expression like $5^3 \times 5^4$, it can be very tempting for them to multiply the indices and obtain 5^{12}, instead of adding them to obtain the correct answer, 5^7. This lesson helps students to see why adding the indices for multiplication and subtracting them for division makes sense.

THE DIFFICULTY

This task is intended to bring to the surface students' difficulties with multiplying and dividing indices.

Match up any cards that are equal to each other. Some cards might not match up with any of the others.

$10^3 \times 10^4$	$\frac{10^9}{10^3}$	10^{12}
10×10^6	$10^3 \times 10^9$	$\frac{10^{14}}{10^2}$
10^6	10^7	10^3

The cards should end up in 4 groups:

$10^3 \times 10^4$ 10^7 10×10^6	10^{12} $10^3 \times 10^9$ $\frac{10^{14}}{10^2}$	10^6 $\frac{10^9}{10^3}$	10^3

If students finish early, ask them to make up two more examples for each of their groups.

Students are likely to mis-match $10^3 \times 10^4$ with 10^{12}, mis-match 10×10^6 with 10^6, mismatch $\frac{10^9}{10^3}$ with 10^3, and mis-match $\frac{10^{14}}{10^2}$ with 10^7. Asking students to check by evaluating each expression (the multiplications by 10 should be easy to do without a calculator) should help them to spot the errors. At this stage, it is enough to get these errors out in the open.

THE SOLUTION

1. Multiplication requires addition of the indices

What does 10^3 mean?

Students might say 10×3 or $10 + 10 + 10$, but this is wrong.

Write down $10^3 = 10 \times 10 \times 10$ and ask students what this is equal to.

Now do the same for 10^4.

Students will write $10^4 = 10 \times 10 \times 10 \times 10$.

Explain to the person next to you what $10^3 \times 10^4$ must be equal to and why. Make up two more examples.

Students will see that $10^3 \times 10^4 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^7$, not 10^{12}.

2. Division requires subtraction of the indices

Now think about $\frac{10^9}{10^3}$ in the same way, by writing out all the 10s. Make up two more examples.

Students will work out that

$$\frac{10^9}{10^3} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times \cancel{10} \times \cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times \cancel{10}} = 10^6, \text{ not } 10^3$$

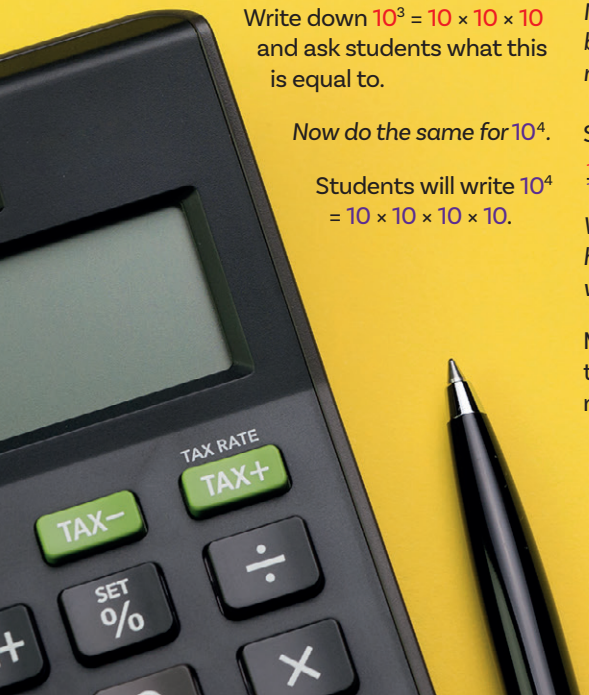
Write a summary in your book of how multiplication and division work with indices.

Make sure that students appreciate that the 10 could be **any** number, but it must be the **same** number throughout

if we are going to be able to simplify the powers like this.

Checking for understanding

Make up a task like the one we started with, with 9 expressions involving indices that have to be matched up. Make sure there are some tricky ones that could catch people out!



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