

## [ MATHS PROBLEM ]

# VARIANCE AND STANDARD DEVIATION

Students are often confused about the meaning of variance and standard deviation, says **Colin Foster**

In this lesson, students use a visual representation to make sense of variance and standard deviation

## THE DIFFICULTY

Two small classes of students took the same test, and these are their marks:

Class 1: 5, 7, 9, 13, 16

Class 2: 1, 7, 9, 15, 18

What's the same and what's different about the marks from these two classes?

Students might notice that there are 5 students in each class, that their medians are both 9 marks and that neither class has a mode. They might calculate the means, and find they are both 10 marks.

The range for Class 1 is 11 marks, while for Class 2 the range is 17 marks – but does this mean that Class B's marks are more varied?

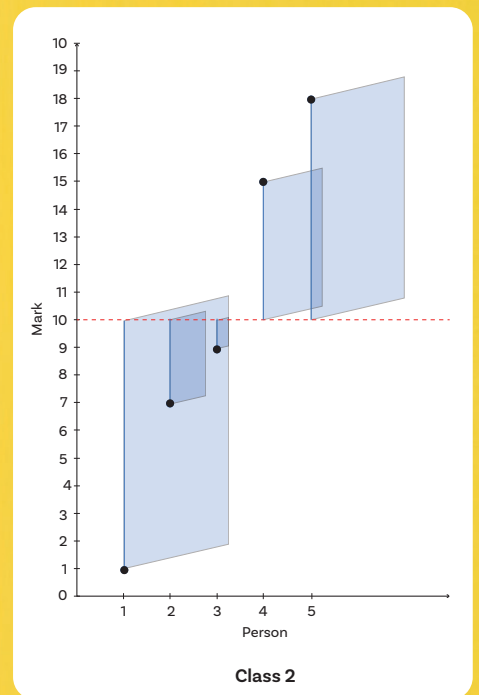
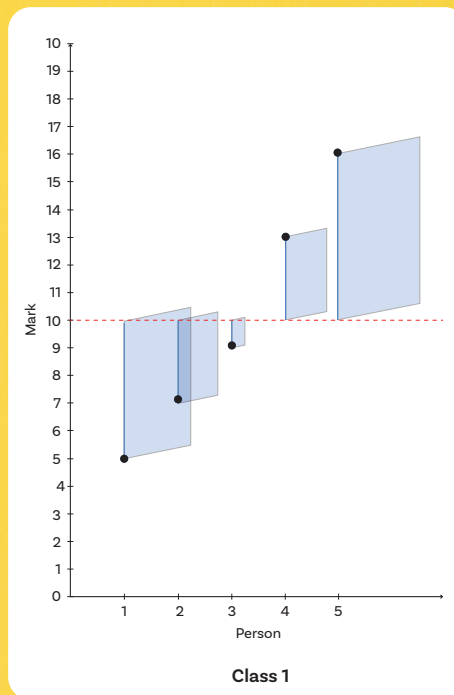
## THE SOLUTION

The trouble with judging variation by using the range is that its value depends entirely on the highest and lowest values in the data set. These two values are, by definition, atypical. Variance provides a way to measure the amount of variation by taking account of **all** of the data values.

We can visualise the deviations from the mean (10 marks) by using blue vertical lines, as shown in the figures above.

The deviations look larger for the second class, meaning that the points are, on average, further from the mean.

But the sum of these (signed) deviations will be zero, because the mean balances the positive and negative deviations perfectly for any data set.



If, however, we **square** the deviations, they will all become positive, as also shown in the figures above.

The **mean squared deviation from the mean** is the **variance**. And its positive square root is the **standard deviation**.

### Checking for understanding

Work out the variance and standard deviation for our two classes' marks.

Students should find a variance of 16 and standard deviation of 4 for Class 1, and a variance of 36 and standard deviation of 6 for Class 2. Class 2's marks are clearly more varied than Class 1's.



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