

[MATHS PROBLEM]

WHEN DO FRACTIONS SIMPLIFY?

When calculating with fractions, students are sometimes unsure when to expect the final answer to simplify and when not, says **Colin Foster**

In this lesson, students explore fraction addition systematically to see when the sum can and can't be simplified

THE DIFFICULTY

If you add two simplified fractions, using their lowest common denominator, will the sum require simplifying or not?

It may take students a moment to work out what your question is asking. If they are unsure, they should think of some examples and try them. They will find that the answer is that the sum will **sometimes** need simplifying but **not always**.

For example, $\frac{1}{2} + \frac{3}{10} = \frac{8}{10}$, which needs simplifying to $\frac{4}{5}$.

However, $\frac{1}{4} + \frac{3}{10} = \frac{11}{20}$, which does not need simplifying.

What determines whether the sum needs simplifying or not?

THE SOLUTION

This task gives students the opportunity to explore this: Use any six different numbers from 1 to 12, once each, in the boxes below, to make as many true statements as you can:

$$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$$

How many solutions are there in which **all** the fractions are in their lowest terms?

What happens if you change the operation from addition to multiplication?

Students will need to do a lot of trial and error, and this will involve them in noticing when they obtain fractions that are already simplified and when they don't.

Up to commutativity, there are six solutions for addition:

| | | |
|---|---|--|
| $\frac{1}{2} + \frac{3}{10} = \frac{8}{10}$ | $\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$ | $\frac{1}{4} + \frac{7}{12} = \frac{5}{6}$ |
| $\frac{1}{6} + \frac{3}{4} = \frac{11}{12}$ | $\frac{1}{6} + \frac{7}{12} = \frac{3}{4}$ | $\frac{1}{12} + \frac{3}{4} = \frac{5}{6}$ |

For multiplication, there are eight solutions:

| | | | |
|---|--|---|--|
| $\frac{2}{9} \times \frac{3}{4} = \frac{1}{6}$ | $\frac{2}{9} \times \frac{3}{8} = \frac{1}{12}$ | $\frac{3}{8} \times \frac{4}{9} = \frac{1}{6}$ | $\frac{3}{10} \times \frac{5}{6} = \frac{1}{4}$ |
| $\frac{3}{10} \times \frac{5}{9} = \frac{1}{6}$ | $\frac{3}{10} \times \frac{5}{12} = \frac{1}{8}$ | $\frac{5}{6} \times \frac{9}{10} = \frac{3}{4}$ | $\frac{5}{12} \times \frac{9}{10} = \frac{3}{8}$ |

Checking for understanding

What patterns did you see?

In the additions, when the denominator of the simplified answer is the same as the lowest common multiple of the denominators of the two original fractions, the final answer doesn't need simplifying.

This happens twice among our six solutions (shown in orange above).

In the multiplications, if we cancel down fully on the left-hand side, the product we obtain will **never** need simplifying.



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