

Knowing Your Limitations

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Summary

This article reflects on whether probability statements can usefully be made about one-off exceptional events.

At a recent performance of the musical *Chicago* (Cambridge Theatre, London), my ears pricked up at a statistical clanger. Roxie Hart fears that she will face execution for her crimes, but is reassured on rather dubious grounds by Mama Morton:

Relax. I mean, in this town, murder is a formal entertainment. Besides, in 47 years Cooke County never hangs a woman yet. So even it's 47 and 1, they won't hang you. (from <http://www.stlyrics.com/lyrics/chicago/fullscript.htm>)

This is a not uncommon kind of argument, and deceptively plausible. Surely Roxie is entitled to draw some comfort from those 47 years of restraint? If it had been only a year or two since a hanging, she would be right to feel more nervous, wouldn't she? Or perhaps not; maybe the opposite is the case – that long period of forbearance, as tension builds, makes it *more* likely that a victim will now be sought? The same dilemma appears when you throw a coin three times and get heads–heads–heads. The gambler's fallacy says that we are 'due' a tails by now, but the coin has no memory and owes no debts to anyone. The opposite point of view says that the evidence for this being a biased coin is beginning to build up, and therefore heads is *more* likely than tails every time with this coin, including on the next throw. How can you decide which line to take?

Whatever the truth of the matter, 47 : 1 cannot be the right odds for Roxie. On this basis, if there had been a hanging the *previous* year the odds would be 0 : 1, which would be absurd! We know nothing of what happened further back than 47 years, or what might account for the cessation, or what the relative seriousness of the various cases might have been. Is probability theory actually relevant at all here?

I don't know how you do calculate Roxie's odds given this information – I suspect that in fact there

is nothing very helpful that you can say statistically. But I think this is an important point: that often with real-life situations (if you'll pardon the term) it really doesn't make sense to try to put numbers to events, and learners of statistics need to realize that this is the case. A pupil whose only contact with statistics is answering textbook questions in school is likely to come to believe that every problem has a neat answer; that if you're given a number you must be able to use it somehow to produce a result. How many textbook problems lead to the answer 'no answer' or 'insufficient information'? Very few – and even fewer are intended to!

One-off exceptional events, such as Roxie's trial, are not really amenable to probability calculations. As we are often told about the financial markets, the past is not a reliable guide to the future. It's like trying to persuade someone that the world is going to end and the Messiah is going to come – all their past experience is against such a thing:

You must understand that in the last days scoffers will come... They will say, 'Where is this "coming" he promised? Ever since our fathers died, everything goes on as it has since the beginning of creation.' (Bible: 2 Peter 3: 3–4, New International Version)

Sometimes people are encouraged by a long period of safety (in proportion, perhaps, to the amount of time since the last disaster), hoping that the danger may have gone for good. Sometimes, for example with natural disasters such as hurricanes and floods, everyone knows that they will come again sooner or later. It is 64 years now since Vesuvius last erupted, so there are plenty of reports that we could be 'about due for another one' in the not too distant future. (Sometimes 'regression to the mean' is explained in a way that reinforces a gambler's pay-back fallacy.)

The big problem here is philosophical. Hume argued convincingly that the idea of a ‘uniformity principle’ (saying something like ‘the future will resemble the past’) cannot be justified rationally. Because something has happened many times before, why should we assume it will happen again – or assume that it won’t? Scientific ‘laws’, even such familiar ones such as the law of gravity, do not *make* things happen in the future – they are descriptive, not prescriptive – they record and summarize how things *have happened in the past*. They have no power in themselves to control the behaviour of the universe. Science is built on the assumption that if you recreate exactly the same conditions, the same observations will recur. But you can never test that because you can never do it; not just because of insurmountable practical difficulties of precision, but because if you consider ‘time’ to be one of the conditions (as modern science insists we must) you cannot reproduce *that!* Furthermore if we argue that science ‘works’,

i.e. that ‘the future has resembled the past’ in the past, and therefore that it will continue to do so in the future, this is, as Hume pointed out, a circular argument.

History may repeat itself or it may not; sometimes it will and sometimes it won’t. We cannot attach precise probabilities to events such as those I have described. Often, the more knowledgeable person realizes that they know less; the better statistician is aware of the virtue of being circumspect. Statistics education should, among other things, lead to a tentative, questioning and critical approach. When younger pupils ask, as they so often do when facing probability topics, questions like ‘What’s the probability that I’ll be the first person from my school to become an astronaut?’ it seems to me that it is time to say that probability can’t help you with such problems. It depends on you, not on probability. Part of statistical awareness is knowing when it is *not* the tool for the job.

STAT MAZE SOLUTION

1 A	N	A	L	Y	S	I	S 2	I	G	N 3
9 E	X	H	A	U	S	T	I	V	E 10	U
G	15 Y	A	T	E	S 16	T	E	M 17	R	L
R	A	21 D	O	U	B	L	E 22	O	R	L 4
A	W	O	A	N	K	25 Z	X	D	O	I
8 L	O	O	24 R	E	W	O	P 23	E	R 11	N
I	W	20 G	N	I	K 19	N	I	L 18	U	E
A	14 T	F	E	L 13	A	M	R	O	N 12	A
7 T	N	E	M	O	6 M	O	D	N	A	R 5