MATHEMATICAL FLUENCY *WITHOUT* DRILL AND PRACTICE

Colin Foster asks how can we avoid letting ‘practice’ dominate the teaching of the new mathematics national curriculum

Introduction

The word ‘practice’ appears twice in the short ‘Aims’ section of the *KS3 Programme of study* (DfE, 2013). The first stated aim is that all pupils:

*… become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately*. (p. 2)

This optimistic sentence implies that focusing on fluency will lead eventually to conceptual understanding and confidence in applying the knowledge gained. This reminds me of John Holt’s (1990) observation that:

… the notion that if a child repeats a meaningless statement or process enough times it will become meaningful is as absurd as the notion that if a parrot imitates human speech long enough it will know what it is talking about. (p. 193)

There is no doubting that the team who drafted these statements worked extremely hard to make them much better than they might have been, and the inclusion of ‘*varied*’ and ‘*increasingly complex problems’* is a heroic attempt to save the sentence from consigning students to an eternity of drudgery. Yet, the fact that fluency focused on ‘*practice*’ is the first stated aim of the programme of study is bound to convey the message that this is something that needs to be got out of the way before ‘*reason mathematically’* and ‘*solve problems*’, the other two aims, can be addressed.

The second occurrence of ‘*practice*’, on the same page, in a paragraph describing how teachers might make decisions regarding students’ progression ‘*to the next stage*’, offers us no reassurance. This paragraph advises that students who ‘*grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4’*, which I very much agree with. However, the following sentence, that ‘*Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on’*, sounds to me like a recipe for never-ending, low-level, imitative rehearsing of knowledge and skills until students earn the right to anything more stimulating.

It is easy to see how students can become trapped in tedious, repetitive work, endlessly ‘practising the finished product’ (Prestage and Perks, 2006). Teachers are going to be told that certain students ‘need more practice on X’ before they are ‘ready’ to move on. Students will be discouraged and demotivated by constant, unimaginative repetition and the low, or slow, achievement that has led to this judgment becomes a self-fulfilling prophecy. What do we do? It is all very well for articles in *MT* to suggest rich, exciting alternatives to mechanical procedural practice, but the danger is that some of our students will never be deemed ‘ready’ for that!

Mathematical etudes

Perhaps what is needed is a way to *disguise* rich, exploratory tasks as though they are merely practice! I am suggesting a bit of underselling here. I want to advocate tasks which look, at first glance, as though they are simply an easy way to generate some routine practice, but which if you dig under the surface have something a little more interesting going on (Andrews, 2002). I have called this type of task a *mathematical etude* (Foster, 2013), by comparison with the genre of ‘etude’ in music.

Musical etudes began as technical exercises for private practice, but are best known today as complete compositions exploring a particular technical problem in a satisfying way. They are far more interesting than practising scales! The later musical etudes, such as Chopin’s, aim simultaneously to please an audience and be an effective teaching tool. They usually focus on one specific technique, and, in the best ones, this self-imposed constraint brings out the creativity of the composer and leads to an elegant, and beautiful, piece of music. At the same time, an etude offers disciplined practice of an important technique. So I use the term *mathematical etude* to refer to a mathematical task that embeds extensive practice of a well-defined mathematical technique within a richer, more interesting mathematical context (Foster, 2013). Of course, this is not a new idea (Andrews, 2002), and many well-known tasks could be placed in this category!

Connected expressions

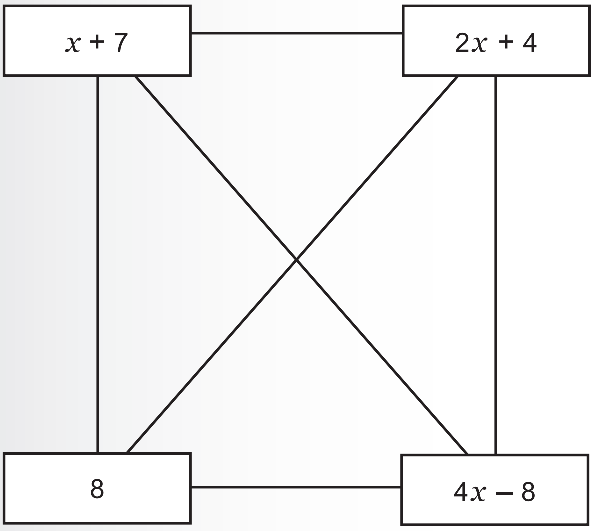

Figure 1. Four connected expressions (Foster, 2013)

One example of a mathematical etude would be 'connected expressions' (Foster, 2012, 2013). The diagram in Figure 1 shows four algebraic expressions with every expression joined to every other expression by six lines. Next to each connecting line you write the solution to the equation formed by equating that pair of expressions. So, for example, the top line joining $x + 7$ and $2x + 4$ corresponds to the solution to the equation $x + 7 = 2x + 4$, which is $x = 3$, so you write 3 next to this line. This means that giving a diagram like this to students is equivalent to asking them to solve six linear equations. However, there are some important differences, leading to thinking that is above the level of rehearsing procedures:

- The six equations produced are connected, arising from just *four* expressions, rather than 12. Why do four expressions lead to six equations? What would happen with a different number of starting expressions?
- Students can begin with whichever pair of expressions they wish, and may feel that they are creating, or revealing, the equations rather than simply being given them.
- There is a pattern to the solutions obtained. If a student gets one solution wrong, it will stick out, and they may well be inclined to check it.
- The pattern {1, 2, 3, 4, 5, 6} in the six solutions is designed to be provocative. When students find the six solutions, they typically say, '*That's cool!*' or '*How did you do that?*' Students may initially think that it is easy to make up one of these themselves. This then becomes the main task.

When students create their own 'connected expressions' diagram, choosing which expressions to use entails trial and error, in other words 'practice', where they pick expressions and then solve the resulting equations. In time, this is likely to develop into 'intelligent' trial and error, in which students unpick how the process is working so as to achieve what they are aiming for. They might aim for a solution set such as the prime numbers {2, 3, 5, 7, 11, 13}, but actually even just aiming for integer answers can be a demanding starting point! Students can also vary the number of expressions, and a triangle of three expressions is a good place to start. Students can obtain a considerable amount of practice from this task while interesting thinking is also going on. For example, one Year 8 student who was aiming for the set of solutions {2, 4, 6, 8, 10, 12} carefully doubled each expression, and was surprised to find that his solutions came out exactly the same as for the original diagram! This led to interesting discussion and further exploration.

My point is that I think you can sell a task such as this as 'practice of solving equations' while smuggling in some rich mathematical thinking. Students will differentiate the task for themselves, choosing easier/harder, fewer/more expressions, and are free to make the problem more/less demanding as they go on. You can even make the expressions quadratic if you want additional challenges! The aim of developing students' fluency in solving equations is supported by the way in which the task attempts to entice them away from the nitty-gritty of the process and onto the higher level of which expressions to choose and why. This 'distraction' helps students to learn to carry out the process without having to attend to every detail, allowing them to find that they can focus on the bigger picture. At the same time, the problem gives students who are already competent at the technique something more interesting to think about. It turns a routine exercise into a self-differentiating, rich mathematical problem – at least that is the intention!

Conclusion

Fluency with important mathematical processes is not a bad thing. I agree with Nel Noddings (2003) that *'Drill should be used judiciously – to routinize skills that will make the learning of important concepts easier and more enjoyable'* (p. 123). I want students to be confident at manipulating algebraic expressions and solving equations. If they are going to move on to more advanced mathematics at A-level, for instance, they are going to be held up and frustrated if they cannot solve equations like these quickly and easily. I would feel that I had let students down if I did not give them opportunities to develop such skills. So, I see fluency as a legitimate goal, but I do not want fluency to be central, and I certainly refuse to withhold rich mathematical tasks until 'basic' skills such as these have been mastered. With a mathematical etude, students gain practice while engaging in a rich problem-solving context.

Even teachers committed to ATM principles, and the value of rich tasks, will often accept the need for 'boring' lessons from time to time, where students 'just practice' something. However, if we could find enough examples of mathematical etudes then perhaps we would not need to settle for any of these 'exercise' lessons? Students could finally stop calling the books they write in 'exercise books'! Mathematical etudes might also be a way to deal with the situation where you are required to teach students to be fluent in something that you don't actually think is necessary or important. ICT is redefining the kinds of mathematical fluency that are relevant: fluency in looking up values in log tables, for instance, is clearly unimportant to anyone today. Maybe also we need to rethink the value of students solving equations by hand? Actually I would still defend that strongly in terms of the mathematical ideas involved, but there are other techniques, such as long division, which I find it much harder to defend spending classroom time practising, and yet which are now explicitly written in to the new curriculum. Unless teachers are prepared for their students to be disadvantaged in high-stakes tests, they are going to have to help them develop fluency in long division. How do we do this without sacrificing our principles – and students' sense of mathematics as a meaningful subject? Perhaps by developing a mathematical etude for long division?

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