Knowing and not knowing how a task for use in a mathematics classroom might develop

Colin Foster, Mike Ollerton and Anne Watson discuss

Participants at the July 2014 Institute of Mathematics Pedagogy (IMP14) engaged in a wide range of mathematical tasks and a great deal of pedagogical discussion during their four days last summer. Towards the end of IMP14 a conversation began regarding how much knowledge about a task a teacher needs to have before feeling comfortable taking it into the classroom.

The conversation …

Colin
If you're planning to use an open mathematical task with learners, then how do you prepare? An open task may lead learners in all sorts of different directions. Should you try to explore every possibility yourself before you go into the classroom? With an extremely open task for example, *"What can you tell me about this (A6) sheet of paper?"* it would be impossible to envisage everything that might arise and prepare in detail for it! Does this mean such tasks are best avoided? Or is that freedom important, so learners can be creative and you can be surprised by what they come up with?

Mike
I think it would be useful to qualify terminology so we are clear about definitions. This is because your conception of an 'open mathematical task' might be different to mine.

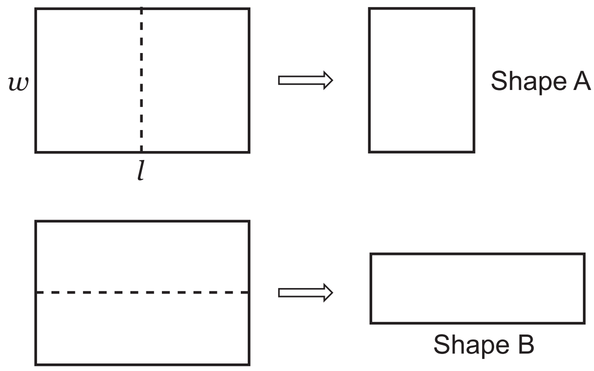
Colin
I think I would see an open task as one that has multiple intended outcomes. I enjoy working on mathematical tasks, so any task interesting enough to be worth using with learners is likely to be something I would want to explore myself. If the task doesn't grab me, why should I expect anyone else to want to do it? But a really open task will be inexhaustible, so at some point in my preparation I will have to decide that I feel "ready" to run with it, even though I can't possibly know everything there is to know about it. So I'm thinking about how much you need to know before you feel ready to share the task with a class.

Mike
I think there are two aspects about what I want to know prior to offering learners a task; these are:

- where the task might fit into an overall scheme of work;
- what developments I might offer arising from the initial task.

A good example is to consider the earlier question: *"What can you tell me about this (A6) sheet of paper?"*

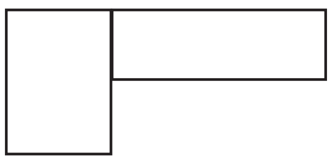
Past experience tells me I will gain a wide range of responses ranging from: area, perimeter, angles, colour, thickness (therefore it is actually a very thin cuboid) and I can choose where to go next. However, I may well have already chosen the next stage and this could be to produce a second, different coloured piece of A6 paper and fold each piece in half, one bisecting the short sides and the other bisecting the long sides:



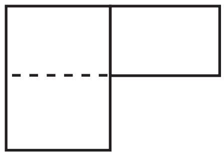
There are now different question I can pose and these are:

i) Prove which of shapes A and B has the greatest perimeter?

ii) Make some compound shapes using one each of shapes A and B and find how many different perimeters are possible in terms of w and l for example:



iii) Make some overlapping shapes and calculate areas and perimeters in terms of w and l.



Clearly there are opportunities here for learners to use and apply algebra and engage with proof.

Colin
That's really nice! Now, it's perhaps not too hard to prepare for part (i). I can see that $2l + w > 2w + l$ if $l > w$, and I can imagine having lots of nice conversations with learners about that and maybe encouraging them to formalise it a bit. I can imagine a bit of measuring with a ruler to check conjectures and then trying to develop reasoned arguments. Parts (ii) and (iii) feel more open, and I feel that I need to do a bit of exploring myself to see what possibilities there are. I've been writing down inequalities between $\frac{1}{2}w < \frac{1}{2}l < w < l$ and thinking through combinations of sides that could join and how each possibility would reduce the perimeter.

When I started teaching and used open problems like this and was circulating around the classroom I used to carry around with me sheets of my own work on the task, so I could "check" what the learners were doing. But as I gained experience I felt less and less need to do that because I tried to make it the learners' job to explain to me what they had done and convince me why I should believe that their conclusions were correct. The authority wasn't my previous work on the task but their arguments. That didn't mean that I was approaching learners completely blank, because I remembered a lot of what I had done, but somehow I was less worried if I didn't have all of the details. If they said *"I can get a perimeter of such-and-such"*, my reaction wouldn't be to hunt through my notes to see whether I agreed that their solution was possible – instead, I would ask, *"How did you do it?"* , or *"Can you show me?"*

Mike
There is, I believe, much value in causing learners to go deeper with their mathematics by asking these kinds of questions, and I would want to encourage this as much as possible. There is also something here about experience which a new teacher, by definition, won't have. So how could a new teacher feel sufficiently confident to use such a task for the first time to develop experience? I believe Heads of Mathematics (HoMs) and senior leaders have key roles to play in terms of supporting new teachers to take risks. HoMs are, by dint of their promoted status, more likely to have a 'gut' feeling for the value of trying out an open task so they can support a new teacher by discussing the appropriateness of using it with a class. Part of this appropriateness depends upon the culture within a department and whether, or not:

a) … students are used to engaging with problem solving type tasks;

b) … the quality of the schemes of work with regard to how they are populated with problem solving tasks;

c) … colleagues are encouraged to take risks by trying something new.

This last point is about teacher autonomy; without which how can we cede responsibility to learners?

Colin
I think there is sometimes a bit of mystique over working in these sorts of ways with learners – as though it is extremely difficult and something only the most "advanced" teachers should attempt. I believe all teaching, in whatever style, is extremely hard to do well, but I think the best way to get good at anything is to do it a lot! Sometimes student teachers say they first want to develop a more traditional style of teaching, which they think will be easier, and then, when they have experience, try using more open tasks. The trouble with this, I think, is that the experience they gain by working in a telling-and-explaining mode may not be very useful for working with open tasks. I think sometimes teachers with less experience can be much better at using open tasks, even if they haven't experienced this approach themselves as learners, than experienced teachers who are going through the process of trying to change their ways of working by unlearning instinctive behaviours.

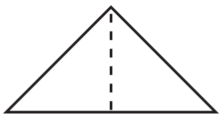
This issue of how to prepare for using open tasks is a big one. For me one aspect is being used to thinking mathematically about problems, not being afraid of being stuck, talking about the difficulties, not just the solutions, and so on. Another aspect is the pedagogical skills of asking good questions, listening to learners and drawing ideas out, taking their ideas seriously and encouraging them to develop them, and so on. And a third aspect is the specific detailed mathematical knowledge involved in the particular task. But all this seems to be asking a lot of a teacher who just wants to make a start! Can we prioritise these things at all?

Mike
The three aspects you raise about effective practice are, I agree, incredibly important. Furthermore, the issue of prioritising time for developing practice is a fundamental component of effective teaching; to cede ever greater responsibility to learners to enhance independent learning. I return to the support roles senior leaders are responsible for enacting; noting a telling statement from the

'Made to Measure' Ofsted publication which reads: '*For senior leaders whose specialism is not mathematics, gaining an understanding of what the best mathematics education involves presents a significant challenge*'. (2012, para 128). I assume this was written because Ofsted had collected evidence to the contrary. Paragraph 129 reads: *'The best leaders inspired and supported teachers in teaching pupils to solve problems and think for themselves in mathematics'.*

I remember an incident about ten years ago when a trainee teacher asked for some advice to help plan a lesson about fractions. I asked if she had considered a task I had previously offered the group about folding and cutting an isosceles right-angled triangle (IRAT), which works as follows:

Fold an IRAT down its line of symmetry then take a pair of scissors and cut it along the line of symmetry to produce two congruent IRATs, each one being similar to and half the area of the original. The diagram below illustrates this, with the dashed line being the line of symmetry through which the original triangle is cut.



The problem develops as follows:

Fold a new IRAT in half down the line of symmetry and then a second time along the 'new', line of symmetry and then make a cut so a square of area ½ and two smaller IRATs of area ¼ are formed. Before revealing the result students could be asked to visualise and describe what shapes are formed in terms of name and area. It might be appropriate to use this to discuss ideas of 'similarity' and 'congruence', thus providing an in-context opportunity to engage with essential vocabulary. With three folds and a cut, down the last fold line, the following shapes are formed: two IRATs of area $^1/_8$, one IRAT of area ¼ and a rectangle of area ½.

Anyway the trainee went away quite happy about planning to use the idea. The following week I asked how her lesson had gone to be met with the response that her mentor did not want her to do the task because he only wanted her to teach the class fractions!

Colin
Yes, I have come across that sort of thing happening! Your IRAT task is focused on teaching fractions, but it's also bringing in geometry and making those connections. I would say that's great, but some teachers, not just senior managers, would think that it's complicating things unnecessarily. They worry it's going to take longer, the students will get confused with all the different ideas, they won't have the necessary background, and at the end no one will be sure what they were supposed to have learned. So, they prefer something more tightly defined. I think we have failed to persuade some teachers that working with open tasks is even feasible in the current educational climate, let alone preferable. There is messiness in working with open tasks that some teachers find really unappealing, and even if you want to work like that it can be very difficult to defend what you are doing to senior managers or Ofsted, so it all feels just too risky. It's much easier to prove that you're doing your job properly if you can explain how you are going in a straight line from A to B – I just don't think people really learn mathematics like that. Can we defend a lesson where we perhaps have a plan like this for starting off something interesting using some kind of prompt, but where the rest of the plan is to "*see what happens*" and "*take it from there*"; to be flexible and adapt to what students come up with? And where we anticipate different students taking quite different things from the lesson, perhaps relating to quite different topic areas?

Mike
Hmm … I wonder if this is where our philosophies differ? I certainly do not intend students to engage with different content knowledge by using these types of tasks. Indeed, with the first one I expect students to engage with central concepts of perimeter and to use and apply algebra. Should some students take the task further, perhaps by having one of shape A and two of shape B, or vice versa, and begin to think about how to prove they have found all possible compound shapes; this fits hand and glove with my conception of differentiated learning. Similarly with the IRAT task – some students are likely to extend the work to consider types and number of shapes from numbers of folds which become impossible to perform physically. The tasks I use are all planned in terms of accessibility relating to the minimum expected level of conceptual understanding I want all students to achieve whilst expecting some students to take ideas further.
So, different students will process their knowledge and engage with problem solving skills for example, to explore, to reason, to conjecture, to generalise, and to prove at different depths. But, as a minimum I intend that all learners develop further their understanding of fractions.

As far as what Ofsted are looking for I return to *Made to Measure* and a statement from the key findings:

Schools were more aware than at the time of the previous survey of the need to improve pupils' problem-solving and investigative skills, but such activities were rarely integral to learning except in the best schools where they were at the heart of learning mathematics. Many teachers continued to struggle to develop skills of using and applying mathematics systematically. (2012, p. 9)

This is, I suggest, a strong statement of intent by Ofsted; a belief of what is important about using problem-solving tasks and encouraging students to simultaneously develop investigative skills. Surely any mathematics teacher, or senior leader, would want to encourage such ways of teaching and learning mathematics... wouldn't they? The fact that such ways of working were around, in my experience, in the mid-70s is merely an indication of a return to a pedagogy of mathematics teaching from at least 40 years ago!

Colin
I think you are perhaps saying that although there will be a lot of diversity in what learners do when they approach an open task, there should not be so much diversity in what they learn from what they do? So, a well-designed open task that leads to a wide range of responses will still be targeting a specific area, or areas, of the curriculum. I think this really helps with planning because although you can't predict precisely what learners will do you have a clear idea of what you intend they will take away.

Mike
Yes, I am happy with your interpretation here about not seeking to predict how far different learners will take a common starting task whilst being clear about the content knowledge intended to be learnt. A key component regarding content knowledge is the interconnectedness of concepts and this is where, I believe, open tasks provide such opportunities for learners to see how mathematics is full of connections.

Anne
Yes, the discussion is very interesting. I think the original issue raised at the *Institute* was about the teacher preparing to use such tasks. The tasks that Mike suggests have another feature as well as the *'open-ness'* (I don't find this a very meaningful expression any more). They have the feature of constraint. In each of the situations there are clear guidelines about what to do with the materials and how to interpret what has been done, the language that it is useful to use, the notations it is useful to use, and the mathematical conventions that it is useful to use. A learner who chose not to do that, but to do something completely different, such as measuring the perimeters of IRATs while everyone else is talking about fractions of the form $(\frac{1}{2})^n$ is going to be a little isolated from the general class discussion. What have they gained from the open-ness? If they are lucky, they have a teacher who could spot that the relationships between the successive perimeters are going to be useful in adding breadth and meaning to the fractions that everybody else is using; if they are unlucky they are going to be isolated in the world of approximate measuring. The learners who are folding and fractioning and discussing shape are laying the foundations for geometrical, deductive, reasoning about similarity, fractions, powers of two, ratios of lengths and areas and so on. The learner who is measuring might be providing data to support this more connected nexus of knowledge, but may not themselves be accessing the geometrical, spatial, connections that provide a feel for similarity.

This is exactly where the knowledge and skills of the teacher are at their most important; in what way can the task be presented so that the apparent constraint of insisting that all students follow the initial same path leads to a deep engagement with mathematics that is unlikely to be achieved if students follow their own questions and approaches? It is the job of the teacher to move learners into places that they would not have reached on their own – i.e. not to leave the measurers measuring while the reasoners reason. So, the job of the teacher is not to provide tasks that can be accessed at any 'level' and can 'go anywhere', but to provide structures – such as Mike describes – that embed key mathematical ideas whose manifestations can be explored and discussed and communicated in some agreed, shared, mathematical language by the whole class. I don't see how the teacher can learn to do that without engaging in tasks themselves, and being at some stage in a position where, having pursued their own investigation, they hear somebody else announcing a finding that has not cropped up for them, and thinking '*why didn't I notice that?*' The answer to the question *'why didn't I notice that?'* will tell the teacher a great deal about how to generate curiosity and a desire for connected meaning in their classrooms.

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