How do you describe mathematics tasks?

Colin Foster and Matthew Inglis ask what it means to describe a task as "rich".

The diagram below shows the graph of y = 2x + 5.

There are four whole unit squares 'trapped' between the line and the axes.

Find the number of trapped squares for other graphs.

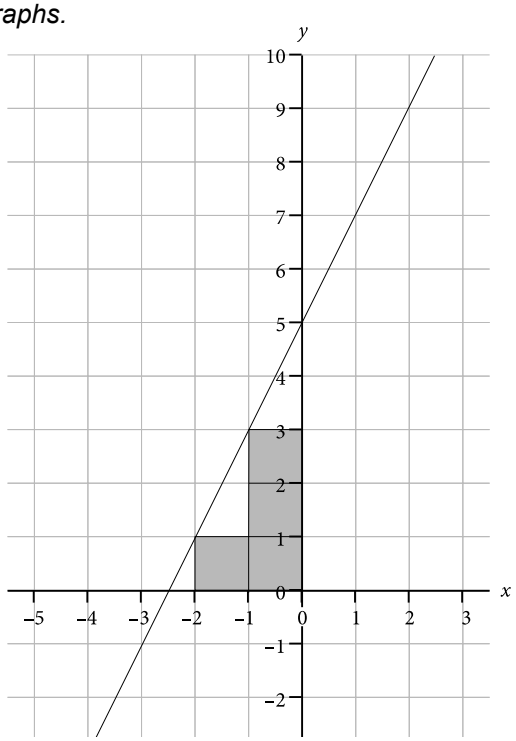

Figure 1: Task taken from Foster (2011, p.185).

What do you think of the task in figure 1? Would you use it with learners? How would you describe it? Many different adjectives are used to describe mathematics tasks, such as "rich", "open", "inquiry-based", "procedural", and so on, but what do they mean? Do teachers understand them in broadly similar ways or in a variety of different ways? The English national curriculum suggests that learners should be offered "rich and sophisticated problems" (DfE, 2014, p.3). But what does that mean? In a recent piece of research (Foster & Inglis, 2017), we carried out two studies to investigate how mathematics teachers use adjectives to describe mathematics tasks.

The two studies

In the first study, we made a list of 84 adjectives that have been used to describe mathematics tasks (you can see the full list in the article, Foster & Inglis, 2017). We then created an internet-based survey where we asked secondary mathematics teachers to think of any mathematics task that they had used recently with learners, or saw another teacher use. They were asked to rate how accurately each of our 84 adjectives described the task. A total of 360 teachers completed the study.

We analysed the data using a factor analysis, a statistical approach that attempts to represent a large number of variables using a smaller set of factors, while accounting for as much of the original variance as possible. This gave us seven factors, which together accounted for 44% of the variance. The seven factors are shown in Table 1, which also shows the adjectives that were most representative of each factor. Table 2 shows the linear correlations between each pair of factors, where 1 indicates perfect positive correlation and −1 indicates perfect negative correlation, with zero indicating no linear correlation at all.

You can see from the small absolute values of the correlations between pairs of factors in Table 2 that the factors were fairly independent of each other, which is what you would hope to obtain from a factor analysis. This means, for example, that how engaging a task was perceived to be was largely independent of how demanding it was perceived to be. There are some weak relationships present, however, which are worth thinking about. Routine tasks were less likely to be engaging than non-routine tasks, as you might expect. You can also see that inquiry tasks were a little more likely to be engaging, but this relationship was also weak, suggesting that there is no automatic link between the use of inquiry tasks and learner engagement. Likewise, tasks which rated highly on the context factor were slightly more likely to rate highly on the engagement factor and the inquiry factor. Tasks which were rated highly on the interactive factor were more likely also to be rated highly on the engagement, inquiry and context factors, although all of these relationships were weak.

The main message is that teachers positioned mathematics tasks along seven relatively

Engagement	Demand	Routineness	Strangeness	Inquiry	Context	Interactivity
enjoyable	difficult	routine	strange	open	real-life	hands-on
fun	complicated	repetitive		inquiry-based	realistic	cooperative
pleasing	demanding	procedural		deep	context-based	collaborative
appealing	perplexing	formal		exploratory	applied	practical
attention-grabbing	easy*	mechanical		investigative		
motivating	challenging	rule-based		rich		
stimulating	simple*			thought-provoking		
memorable	problematic			closed*		
boring*	puzzling			analytical		
interesting						
absorbing						
exciting						
inspiring						
dull*						
engaging						

Table 1: The adjectives most representative of each of our seven factors. (*indicates an adjective that loaded negatively)

independent dimensions, which could be interpreted as meaning that there are seven separate features of mathematics tasks.

In our second study, we looked at whether teachers agreed about how particular adjectives related to the same task. In other words, if one teacher believed that a task was "rich", would another teacher agree? To do this, we had to present teachers with a given task and ask them to rate how well they felt it represented each factor. This time we found that teachers disagreed quite a lot. For example, some teachers felt that the *Trapped squares* task was engaging and inquiry-based, the signature characteristics of "rich" tasks according to our first study, but many regarded it as neither engaging nor inquiry-based. We found more agreement concerning the context factor, but that might simply have been because all of the tasks we used were fairly pure. We did find that teachers interpreted routineness quite consistently, however.

Implications

One way to interpret the seven factors is that if you are choosing or designing a mathematics task, these are seven things that you might want to think about. In other words, do not assume that dealing with one of them will automatically take care of any of the others. For example, engagement and inquiry are perceived by teachers to be only weakly related. So, while an inquiry task might be engaging, it might not. The link between these two features is not strong.

Our results offer a possible reason for disagreement about what constitutes a "rich' task. A task's richness seems to depend on at least two largely independent properties, because the word "rich" loaded strongly onto both the inquiry and the engagement factors, suggesting that richness is a multidimensional notion.

	Engagement	Demand	Routineness	Strangeness	Inquiry	Context	Interactivity
Engagement	1.00	.08	–.20	–.04	.32	.26	.29
Demand		1.00	.02	–.06	.28	.03	.01
Routineness			1.00	.06	–.10	.05	–.13
Strangeness				1.00	.09	–.06	–.13
Inquiry					1.00	.30	.20
Context						1.00	.27

Table 2. The correlations between each pair of factors.

A final implication concerns teacher agreement and disagreement. Teachers were quite internally consistent in their ratings. For example, if a teacher felt that a given task was "appealing", they were also extremely likely to believe that it was "pleasing" (two words from the same "engaging" factor). However, there was little between-teacher agreement. This means that we should not assume that teachers will interpret words like "rich" in the same way. This is a problem, because it limits how effective it can be talking in general terms about, say, "rich" tasks. If you want someone to know what you mean, you really need to give examples and not rely too much on adjectives.

Conclusion

One problem with our study is the relationship between the task, what you ask learners to do, and the activity, what actually happens as a result when you use the task with particular learners. It may even be that "There are no rich mathematical tasks, only tasks used richly" (Mason, 2015, p.15). Does it make sense to try to judge a task in isolation? Maybe it is only sensible to say that a task is "rich" if you are thinking of somebody who finds it so? However, it seems to us that we have to talk about tasks using some kind of language and we need some basis for choosing one task rather than another to use with learners. So, we think that exploring the way that teachers talk about tasks is an important thing to do. We hope that the seven dimensions might help when thinking about designing or selecting tasks to use with your learners.

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