Breaking Free from Convention

This isn't to suggest we abandon horizontal number lines entirely, but rather that we expand our toolkit. Why not use both orientations? A classroom might feature a vertical number line alongside the traditional horizontal one, helping children see that mathematical relationships aren't confined to a single representation.

Some teachers have experimented with diagonal number lines, circular arrangements, or even spiral patterns. Each orientation can highlight different mathematical properties and appeal to different learning styles. A child who struggles with left-right directionality might flourish with up-down movement instead.

Moving Forward

As mathematics educators, we have an opportunity to question our assumptions and consider whether our teaching tools serve all learners effectively. The humble number line, such a fundamental part of early mathematics education, deserves this scrutiny.

Perhaps it's time to turn our number lines on their side - literally. By embracing vertical orientation, we might find that mathematical understanding reaches new heights, one number at a time.

Our students deserve mathematical tools that connect with their natural understanding of the world. Sometimes, the best way forward is simply to look up.

Ray Huntley is almost entirely retired but still enjoys learning and teaching mathematics in any situation, except perhaps when enjoying his beach hut!

Which way should a number line go?

Korbinian Moeller, Dan O'Brien and Colin Foster

explore the merits of vertical number lines

In classrooms where children learn mathematics, one of the things you're sure to see somewhere is a number line. But these number lines can vary quite a bit from one to another – and one difference is which direction the number line goes. Are your number lines horizontal or vertical? Does it matter?

We think that we see horizontal number lines much more often than vertical ones. Perhaps that's simply because we read and write from left to right, which makes it easier for books or worksheets to format them that way round. In fact, reading direction seems to have profound effects on children. Children's early experiences seeing writing going left to right in western cultures seem to create a bias towards a horizontally (left-to-right) oriented number line, even before they start reading and writing themselves (Göbel et al., 2018).

Gravity might be another factor in the predominance of horizontal number lines. A teacher might want to hang number cards on a washing line strung across the room, or laid out along the floor, making a horizontal orientation very natural. Letting children

walk along such a number line on the floor can help them to understand number magnitude (Link et al., 2014). In contrast to these horizontal number lines, a physical vertical number line could entail children climbing onto chairs and ladders, which might be harder to implement or even dangerous.

But are these cultural and practical considerations good reasons for children's number line experiences to be mostly horizontal rather than vertical? We think not. Indeed, we think that there may be good reasons why children's first encounters with number lines should be vertical, rather than horizontal.

Vertical number lines fit natural metaphors and physical experiences

When talking about numbers, we often use metaphors that involve vertical, physical space. For example, children grow up as they become older and they move up to the higher school years. These metaphors reflect natural world experiences, such as that more of something typically piles up higher than less of something. These experiences provide grounding to the abstract idea of number (Myachikov et al., 2014). So, when a child is asked, "Which is more, 3 or 4?" they may turn to these grounding experiences to visualise 4 as being above or higher up than 3. We think that these natural world experiences are better reflected on a vertical number line than on a horizontal one.

Moreover, we can think of larger numbers not just as higher up, but as the result of a journey going up a mountain or traversing some steps or stairs. In these cases, increasing height is not only associated with becoming higher up, but also with increasing physical effort (Link et al., 2014). These bodily experiences seem more concrete than their horizontal counterparts, such as increasing distance travelled in a certain direction. Going upstairs and going downstairs feel opposite in terms of effort – there is a tangible asymmetry that isn't part of the experience of traversing a horizontal number line.

As such, numbers arranged on a vertical number line mesh better with such grounding experiences from our surrounding world, such as things piling up or gravity making moving up more effortful.

Vertical number lines are not bound to cultural conventions around left and right

To use a horizontal number line, you need to remember the arbitrary convention that numbers to the right are greater than numbers to the left – at least in our left-to-right-reading western cultures. It could just as well be the other way round if we had happened to have made our number line start with zero on the right, instead of on the left. Indeed, there is evidence for a right-to-left-oriented number line in cultures that read from right to left (Shaki et al., 2009).

In line with the cultural convention of a left-to-right oriented number line, there is evidence that adults react faster to smaller numbers with their left hand and larger numbers with their right hand (Wood et al., 2008). Adults also tend to associate addition with movements to the right and subtraction with movements to the left (McCrink et al., 2007). However, this association seems to be learned, as the respective effects increase with age and experience with the left-to-right orientation, and so this doesn't seem to be knowledge that's grounded in natural-world experiences.

By contrast, vertical number lines don't require remembering a left-to-right cultural convention or even distinguishing between left and right, which many people find difficult. Many people confuse left and right, but no one confuses up and down! Therefore, we think that a vertical number line avoids left-right cultural conventions which a horizontal number line is completely dependent on.

Vertical number lines make more sense for negative numbers.

So far, we've outlined the potential benefits of vertical number lines for comprehending number magnitude and positive-integer arithmetic. However, we think that the advantages of number lines are particularly apparent when it comes to negative numbers, and here too we think that a vertical number line is preferable over a horizontal one.

It may be that children no longer have everyday experiences with bulb thermometers, where the liquid level rises as the temperature goes 'up'. But we still use the language of 'high' and 'low' temperatures – and also 'below' zero, when working

with Celsius temperature. Moreover, children are familiar with 'negative' height as depth, such as when taking an elevator below ground level or swimming under water. A vertical number line helps us to see that it makes sense to say that –3 is 'higher than' (or 'greater than') –4, even though the magnitude of –3 is less than the magnitude of –4. With a horizontal number line, all of this is much more abstract and arbitrary, depending on remembering that right is 'higher' than left, even though it is in fact physically at the same 'height'.

Conclusion

There is evidence that learners are likely to benefit from encountering number lines whatever direction they are presented in (Leonhard et al., 2023). We certainly don't think that horizontal number lines are harmful! Ultimately, we want learners to become comfortable with both directions of number line – and eventually to combine one of each to create Cartesian axes.

As digital technology is increasingly used for teaching with number lines, we are becoming increasingly free from written or printed page constraints and able to position number lines in whatever orientation we prefer.

Variety and flexibility with number line orientation may be optimal eventually, but we think that varying the direction too soon is a missed opportunity and that consistency with using a vertical number line may be preferable. Switching direction based on where there might happen to be more space on the board or wall, or according to a resource writer's arbitrary preferences, seems sub-optimal. An 'expert', such as a teacher, who has lots of familiarity with number lines, might underestimate how demanding this kind of switching could be for a 'novice' learner.

In current practice, we think that number lines are generally encountered in the order: horizontal, then vertical. And we suggest that reversing this order could build better on learners' developing experiences of the world around them.

References

Göbel, S. M., McCrink, K., Fischer, M. H., & Shaki, S. (2018). Observation of directional storybook reading influences young children's counting direction. Journal of Experimental Child Psychology, 166, 49-66.

McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. Perception & Psychophysics, 69, 1324-1333.

Leonard, S. J., Roche, C., Durkan, A., Gomides, M., & Santos, F. H. (2023). Children grow upwards, and so does the number line: Evidence from a directional number line paradigm. Progress in Brain Research, 279, 37-56.

Link, T., Schwarz, E. J., Huber, S., Fischer, U., Nuerk, H. C., Cress, U., & Moeller, K. (2014). Maths on the mat: Embodied training of basic numerical competencies. Zeitschrift Für Erziehungswissenschaft, 17, 257-277.

Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. Psychonomic Bulletin & Review, 16(2), 328-331.

Wood, G., Willmes, K., Nuerk, H. C., & Fischer, M. H. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. Psychology Science Quarterly, 50(4), 489.

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