ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme A
Lisbon, Portugal, 3-7 February 2020

LESSON STUDY: A CASE OF EXPANSIVE LEARNING

Geoff Wake — University of Nottingham — geoffrey.wake@nottingham.ac.uk
Colin Foster — Loughborough University — c.foster@lboro.ac.uk
Keiichi Nishimura — Tokyo Gakugei University — knishi@u-gakugei.ac.jp

Lesson study is increasingly used throughout the world as a model of collaborative professional learning. In this article we describe the theoretical foundations of an ongoing study which seeks to explore how we might inform collaborative learning with a focus on the development of curriculum coherence. The study involves collaboration between teachers and researchers in both the United Kingdom and Japan. We draw on the theory of expansive learning as a development of Cultural Historical Activity Theory and give insight into the potential this has to inform our work with a range of didactical devices or artefacts that provide connections across topics of conceptual understanding of mathematics and development over time. Expansive learning as a theoretical lens helps us understand our exploratory work in this area and provides insights into how this may be taken forward over the course of the study.

Introduction and Context of the Study

Lesson study is a long-established model for teacher professional development that originated in Japan and involves a process of identifying a professional research question, collaborative lesson planning, observing the lesson, analyzing what takes place, and reflecting on outcomes in relation to this question in a "research lesson" (Lewis, Perry, & Murata, 2006; Lewis & Hurd, 2011). Thus lesson study provides for collaborative professional development that "ties both theoretical and practical learning together in a most authentic way – *through teaching*" (Pothen & Murata, 2006, p. 824).

This article originates from the work of an ongoing study that brings together teachers and researchers in the United Kingdom and Japan seeking to understand how in collaboration (of teachers in each country separately and of researchers between countries) and working within a lesson-study paradigm they might better understand how to develop curriculum coherence. Put simply, we wish to establish how we might gain insight into how to organise teaching to provide students with a more connected experience of learning mathematics. The current focus has been informed by observations and discussions of the carefully designed textbooks used by teachers in lesson study activity in Japan.

In this article we provide insight into the theoretical underpinnings of the study. In particular, we draw on the theory of expansive learning (Engeström, 1987, 2001) to provide insight into how we have considered the work to date and how this is informing the design of future approaches that support our aim to develop professional learning for curriculum coherence. The theory of expansive learning builds on the more general cultural-historical activity theory (CHAT), which is firmly rooted in the Russian school of social psychology originally developed by Vygotsky, Leont'ev, Il'enkov, and Davydov. In the next and following sections, we explicate the ideas appropriate to the study that relate to CHAT and the theory of expansive learning. We provide a brief illustrative case showing how these theoretical ideas might be harnessed to better understand how we might use lesson study to develop coherence in curriculum implementation in mathematics.

Cultural Historical Activity Theory, Boundaries and the Potential of the Role of Didactical Devices

Cultural Historical Activity Theory

Cultural Historical Activity Theory (CHAT) has been increasingly used to make sense of the complex activities that are involved when groups of individuals come together in a group to work with common purpose to achieve some particular outcome(s). The theory provides a useful lens through which to view how both the actions of individuals and the joint activity of the group are mediated by a range of factors in pursuit of individual and jointly-shared goals. Particularly helpful in the case of the study reported here is *third-generation* CHAT, which provides insight into interactions between two or more systems. Consequently, ideas of boundaries, boundary crossing, and boundary objects prove useful (Wake, Foster & Swan, 2013).

Fundamental to CHAT is the Vygotskian notion of goal-directed actions of the individual (Vygotsky, 1978). This considers how the actions of individuals are mediated by *instruments*. It is important to our later analysis that we distinguish carefully at this point between the terms *artefacts* and *instruments*. The word *tool* is often used instead of *instrument*, and this we find unproblematic, but it is important to draw a distinction between the use of *artefact* and *instrument/tool. Artefacts* are devices that are generally available with multiple potential uses, whereas instruments/tools are being used for a specific purpose (Daniels, 2001). For example, the pedagogic strategy of *think-pair-share* is used by teachers in classrooms around the world to facilitate constructivist, dialogic approaches to teaching. However, the teacher's decision to use such a device shifts it from being an available artefact to being an instrument and we consider this as an artefact being made instrumental in its use.

A wide range of artefacts is available to teachers in classrooms, including, for example, diagrammatic representations, texts, presentation slides, manipulatives, and non-material objects such as discourse/language in the sense of Werstch (1991). Our study identifies three important classes of artefacts available to the mathematics teacher: general pedagogies, mathematics-specific pedagogies (i.e., didactics) and what we have termed *didactical devices*. As explained above, we see something like *think-pair-share* as an example of a general pedagogy. An example of a mathematics-specific pedagogy would be the strategy of seeking to draw learners' attention to mathematical structure by asking students in their lessons to give an example that demonstrates understanding of a particular concept and then asking for "another", " and another", "and another" (Watson & Mason, 2006). This particular pedagogic strategy has generality of use in mathematics teaching, but may be of perhaps lesser value in the teaching and learning of other subjects. In this study we are concerned with the construct of didactical devices which are more closely connected with specific aspects of mathematical knowledge and its structuring. We will provide examples of such devices later in this article. It is our emerging belief that this latter category of artefact has the potential to provide an important set of devices that can facilitate teacher learning in relation to teaching for curriculum coherence. Importantly, here we consider the learning of both individual teachers and researchers as well as that of the collaborative collective lesson study research group.

The notion of the goal-directed action of individuals (upper triangle in Figure 1) was expanded by Leont'ev (1981) in second-generation activity theory to include the community in which individual actions are aggregated in pursuit of joint activity. The lower triangles highlight the additional mediating nodes considered by Leont'ev, indicating how ideas of *community*, *division of labour* and *rules* mediate the actions of the subject as an individual in relation to the object of activity of the collective.

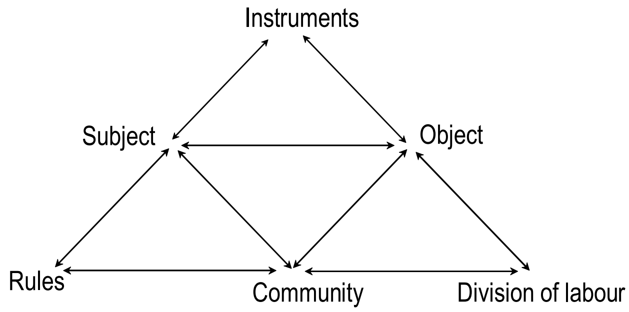


Figure 1: Engström's Cultural-Historical Activity Theory model that introduces mediating factors of the activity system proposed by Leont'ev that brings together individuals in pursuit of a joint enterprise.

There are many rules, both explicit and implicit, that regulate the behaviour of individuals in collaborative enterprise. Important to our study is the joint enterprise of lesson study and the community of teachers and researchers that are brought together in the lesson study group. In this case, the group develops ways of working that over time lead to expectations and rules in relation to all stages of the lesson study cycle: identifying issues, developing a research question, planning the research lesson, teaching and observing the research lesson, post-lesson reflections and discussion and planning future activity. Leont'ev (1981), in his consideration of the activity of the collective, also drew attention to the division of labour between participants, and how a sense of community begins to determine who has agency and control over deciding the direction of the activity. Although in lesson study as a collaborative professional activity we seek to provide a safe space in which the views of all are respected, it is important that some responsibility for group leadership is invested in certain individuals and that they foster a strong sense of community.

Boundaries

As individuals, we are all involved in multiple Activity Systems. Classroom teachers of mathematics, for example, in secondary education may be organised to work collectively in a distinct mathematics subject department. An individual teacher in this activity system is involved in different actions from those that they carry out in other activity systems, such as classrooms. For example, as a member of a mathematics department in a school a teacher may have a role in developing a scheme of work that organises the curriculum for the group of teachers with whom they work, whereas in the classroom they might directly interact with students.

Third-generation Activity Theory (Engeström, 2001) models relationships between the multiple activity systems which individuals inhabit as they work towards the different goals that result from the different overall objects of activity of the different collectives (for further discussion see, for example, Wake, Swan & Foster, 2016). In CHAT, this simultaneous membership of, and participation in, multiple activity systems leads to *boundary crossing* "as a socio-cultural difference leading to a discontinuity in action or interaction" (Akkerman & Bakker, 2011, p. 133).

Boundary crossing is often facilitated by *boundary objects* (Star & Griesemer, 1989), which are artefacts that have different meaning and use in two or more different Activity Systems, while retaining a common essence across systems. Didactical devices are examples of such boundary objects: in the classroom they support the teacher's work of didactical transposition (Brousseau, 1997) as they facilitate building on students' current understanding to develop new conceptual understanding, whilst in the lesson study group they provide teachers with devices that assist their consideration of epistemological issues.

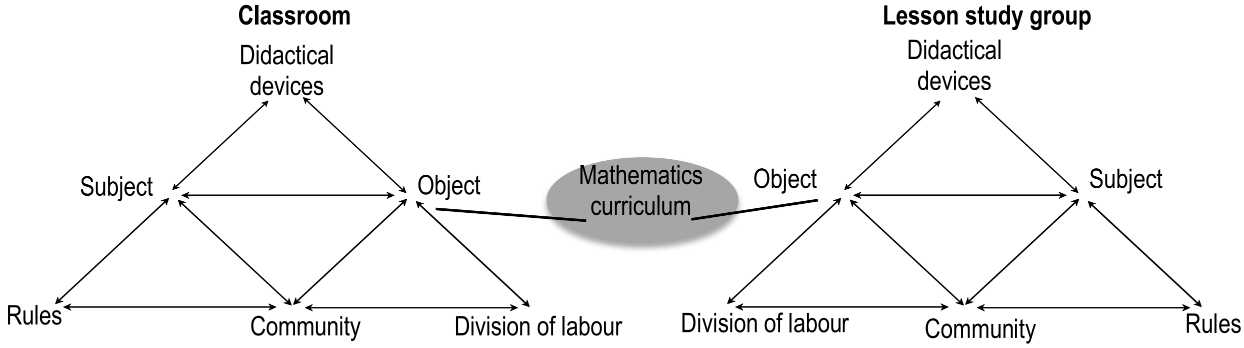


Figure 2: Interacting activity systems of classroom and lesson study group

The potential of the role of didactical devices

The notion of the transposition of 'everyday' knowledge of mathematics into mathematical knowledge for teaching has been acknowledged as an important, and often problematic, issue and explored, primarily in Western education research, for a number of decades (for example, Chevallard (1985)). The most concise and logical way to present a curriculum specification, often referred to as the *intended* curriculum (Schmidt et al., 1997), as opposed to the curriculum as enacted in the classroom, is often far removed from how it might best be sequenced and taught. In particular, the didactic devices that underpin specialist content knowledge for teaching (Ball et al., 2008) are rarely explicit in curriculum specifications. In the UK, current curriculum specifications are primarily lists of content. These are commonly organized, as in many nations, into generally accepted sub-domains of mathematics: number and measure, algebra, geometry, probability and statistics, with many countries drawing on the Organisation for Economic Co-operation and Development's framework that specifies mathematics as a domain of study (2016). The work of teachers then involves a process of didactic transposition (Brousseau, 1997), whereby knowledge as presented in curriculum documents is adapted in ways that make it suitable and sequenced as learning objects.

It is clear, although not explicitly emphasised in the Japanese texts, including student text books and teaching guides, that boundary objects that support didactic transposition are introduced (what here we are calling didactical devices). For example, the number line as a representation is used in increasingly sophisticated manifestations again and again across the curriculum to provide a coherence in approach to support students develop common ways of seeing concepts with which they are familiar and to help support insight into new concepts that are introduced as objects of learning. For example, the development of the number line as the *double*-number line can be used by both teachers and learners to gain insight into their different conceptualisations of proportionality and its effective application in a range of contextual problems. In other work, the UK group has explored how the double number line used in this way as a didactical device can be developed as a boundary object to support teacher learning in a modified version of lesson study. We have found that, in Drijvers & Trouche's terms (2008), such a didactical device becomes an *instrument* that both shapes the thinking of the user, in this case the teacher (the instrumentation process), and is in itself shaped by the user (the instrumentalisation process). From this theoretical perspective, we are considering how didactical devices might inform the process of curriculum design for coherence in ways that allow for teachers to engage in co-construction of knowledge and documentary work as they engage in the process of didactical transposition for effective learning.

Expansive Learning: an illustrative case

In an attempt to capture the multidimensional and complex nature of learning, Engestrom (1987) draws on the theory of expansive learning which "puts the primacy on communities as learners, on transformation and creation of culture, on horizontal movement and hybridization and on the formation of theoretical concepts. Fundamental to this is consideration that through collaboratiion participants construct a new object and concept for their collective activity and implement this in their practice. Engestrom and Sanino (2010), point to a variety of interventions/studies that have drawn on the theory of expansive learning, such as those by FitzSimons (2003) that considered adults learning mathematics in workplaces and learning as boundary crossing in a school-unversity partnership (Tsui and Law, 2007). Important to the theory is that the subject of learning is transformed from the learning of the individual to the learning of the collective in ways that leads to redefinition of the object of activity of the collective. This is perhaps best explained by reference to a specific example arising from the work of the study to date.

Following an intial analysis of the series of Japanese text books, developed over many years by the Japanese research team, and discussions between both research teams, three experimental lesson studies were held in the UK in 2019 and attended by both research teams working collaboratively with the UK teachers. The focus of the lessons was the potential of the construct of vectors as a didactical device to support the development of mathematical knowledge across a range of lessons in different topics in mathematics. Here, we briefly illustrate the use of the notion of "vector" as a didactical device in relation to a lesson on the addition of directed numbers (positive and negative numbers) for students aged 11-12. Important here is how the introduction of the construct of the didactical device, a device that is fundamentally concerned with providing an underlying focus on mathematical knowledge and mathematical knowledge for teaching, expanded the object of the collaborative activity of the planning team, in this case two teachers from one school and the first author as researcher. In lesson study, the object of activity of the planning team is usually a plan for a single lesson that seeks to answer a particular research question by prompting student and teacher activity when the lesson is taught and observed by the wider lesson study group.

We have also had conversations around the idea that the use of a vector representation here is helpful for future learning such as finding the position vector of an object. Specifically, if the position vector of an object at time t is given by the vector **r**, the initial position of that object at time t = 0 is $\mathbf{r}_0$ and the object undergoes a displacement of **s** then $\mathbf{r} = \mathbf{r}_0 + \mathbf{s}$.

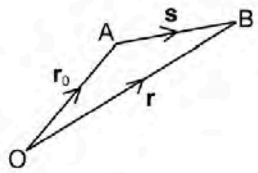


Figure 3: Discussion of vectors in general terms from the teachers' research lesson plan for a lesson on adding and subtracting positive and negative numbers

The activity of the lesson planning group, whilst also focusing on the research lesson in the study reported here, was also informed by the new construct of the didactical device in general, and more specifically the construct of "vectors" in this particular case. This resulted in a new expanded object of activity that focused more than previously on curriculum coherence in terms of how understanding of the underlying mathematical structure of a vector might be used to inform this area of the curriculum (adding and subtracting directed numbers). Whilst the intention is not to study vectors as mathematical objects in this phase the introduction of their underlying properties is found to have potential value as preparation for the future work of students such as when decomposing vectors into horizontal and vertical components in the curriculum, some two years down the line (and the subject of another lesson study in the sequence of three). Figures 3 and 4 show how the teachers' thinking in relation to "vector" as a didactical device was presented in the lesson plan for adding positive and negative numbers. Figure 3 illustrates how the teachers rationalised they would work with vectors later in the curriculum, and Figure 4 provides insight into how their more general thinking about the use of "vector" thinking was transformed to their plan for adding and subtracting positive and negative numbers.

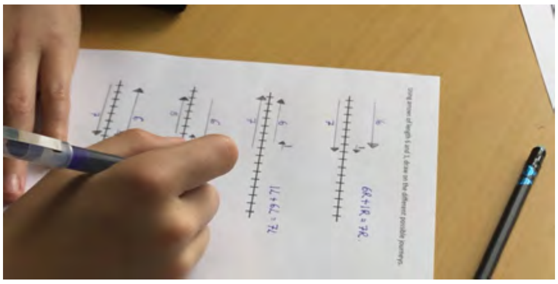
We aim to use vector type representations to first illustrate 'journeys' and then move to representing and solving addition of directed number calculations.

In doing so we aim to draw out the following key points as we progress through the lesson:

- One leg of a journey must start from where the previous one ended (*building towards ALWAYS adding vectors*)
- The destination of a journey can be different for the 'same' journey (*the idea of free vectors*)
- The different destinations for the 'same' journey are dependent on the starting point (*link with the initial position vector and the need for an origin*)
- It is often helpful to have a fixed starting point
- In maths we have a ready-made starting point in zero
- Addition calculations involving directed numbers can be represented using one dimensional vector type diagrams
- Vector type diagrams can help us solve and explain how to add directed numbers

Figure 4: Key points of the teachers' research lesson plan

Figure 5 show some of the work that students undertook exploring "vector style" journeys in preparation for initial work in adding and subtracting positive and negative numbers.



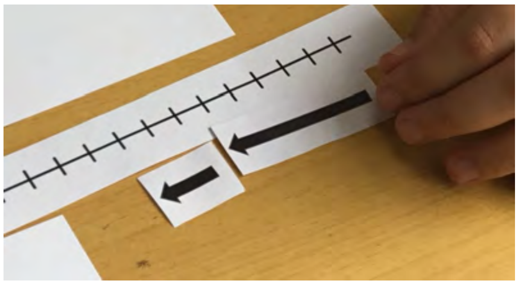


Figure 5: Student work using vector thinking in the early stages of developing conceptual understanding of adding and subtracting positive and negative numbers.

Discussion

Here we have exemplified key aspects of theoretical ideas relating to how the work of teachers collaborating through lesson study might be considered from a CHAT perspective. In particular, we have used the theory of expansive learning, whereby the multifaceted dimensions of learning of both individuals and the collective might be considered as expanding the object of their activity, in order to better understand how we their teaching may better ensure student experience of conceptual connectedness. Our claim is that the introduction of the construct of didactical device as a boundary object with meaning in both classroom and lesson study group has the potential to facilitate such expansive learning in terms of developing teacher knowledge of curriculum coherence. Further, it is our contention that socially distributed teacher knowledge of this type has the potential to improve student learning outcomes as over time they experience mathematics as a conceptually connected knowledge domain. For example, in the brief illustration given here we see students informally working with underlying ideas of vectors that will eventually support their future work where the focus will be on using vectors both explicitly and more formally.

Our introduction of the didactical device into the activity system of the lesson study group we consider as essential in refocusing the work of the group so that the object of study is developed beyond the consideration of a single lesson focused on teaching a particular mathematical concept at a particular time. The didactical device itself seeks to provide a potential tool for teaching (and learning) both across traditional topic boundaries and to support development. over time. Here, we have drawn, briefly, on two such devices: the number line and vectors. Although the latter of these at a later stage becomes an object of study in its own right, here we focus on its potential to provide an underlying structure and way of thinking that can underpin other areas of the curriculum, such as exemplified here: the adding and subtracting of positive and negative numbers. Our work in this area is a work in progress, although we have identified a number of potential other devices, such as the use of units, the circle as a locus of a set of points, the idea of the unit (one-ness).

In activity theory terms, the introduction of the didactical device provides a secondary contradiction, as an old artefact/instrument meets a new object: in the example here, the old artefact of vector, now considered as a didactical device, underpins the new object of the group as it works to better understand curriculum coherence. Contradictions of this type are necessary but not sufficient drivers for expansive learning (Engestrom, 2016). Our theoretical analysis of our work to date suggests that there is much potential for continued collaborative, and expansive, learning facilitated by focussing on further didactical devices. Our analysis highlights the importance of the introduction of the new construct as being central to supporting learning of both individuals and collective as a whole.

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81,* 132-169.

Ball, D. L., Hoover Thames, M., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59(5), 389-407.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Chevallard Y (1985), La Transposition Didactique. Du savoir savant au savoir enseigné, 2nd edn 1991, La Pensée sauvage, Grenoble [Spanish translation: Chevallard Y (1997) La transposición didáctica. Del saber sabio al saber enseñado. AIQUE, Buenos Aires]

Daniels, H. (2001). *Vygotsky and Pedagogy.* Abingdon, UK: RoutledgeFalmer.

Drijvers, P. & Trouche, L. (2008). From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In G. W. Blume & M. K. Heid (Eds.), Research on technology and the teaching and learning of mathematics (Cases and perspectives, Vol. 2, pp. 363–392). Charlotte: Information Age.

FitzSimons, G. E. (2003). Using Engeström's expansive learning framework to analyse a case study in adult mathematics education. Literacy & Numeracy Studies, 12(2), 47–63.

Engeström, Y. (1987). Learning by expanding. An activity-theoretical approach to developmental research. Helsinki, Finland: Orienta-Konsultit.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. Journal of Education and Work, 14(1), 133-156.

Engeström, Y. (2016). Studies in expansive learning: learning what is not yet there. New York: Cambridge University Press.

Engeström, Y., & Sannino, A. (2010). Studies of expansive learning: Foundations, findings and future challenges. Educational Research Review, 5(1), 1-24.

Engeström, Y. (2001). Expansive learning at work: toward an activity theoretical reconceptualisation. *Journal of Education and Work*, *14*(1), 133–156.

Leont'ev, A.N. (1981) Problems of the Development of the Mind. Moscow: Progress.

Lewis, C., Perry, R., and Murata, A. (2006). How Should Research Contribute to Instructional Improvement? The Case of Lesson Study. Educational Researcher, Vol. 35, No. 3, pp. 3 –14

Lewis, C., & Hurd, J. (2011). Lesson study step by step: how teacher learning communities improve instruction. Portmouth: Heinemann.

Organisation for Economic Co-operation and Development (OECD). (2016). PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematic and Financial Literacy. Paris: OECD.

Pothen, B., &Murata, A. (2006). Developing reflective practitioners: A case study of pre-service elementary mathematics teachers' lesson study. In J. V. S. Alatore (Ed.), Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.2, pp. 824-826. Merida, Mexico: Universidad Pedagogica Nacional.

Star, S. L., & Griesemer, J. R. (1989). Institutional ecology, "translations" and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907–39. Social Studies of Science, 19(3), 387-420.

Tsui, A. B. M., & Law, D. Y. K. (2007). Learning as boundary-crossing in school–university partnership. Teaching and Teacher Education, 23, 1289–1301.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press.

Wake, G., Foster, C., & Swan, M. (2013). A theoretical lens on lesson study: Professional learning across boundaries. In A.M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 369–376). Kiel, Germany: PME.

Wake, G., Swan, M., & Foster, C. (2016). Professional learning through the collaborative design of problem-solving lessons. *Journal of Mathematics Teacher Education*, 19(2), 243–260.

Watson, A. and Mason, J. (2006) Mathematics as a Constructive Activity: LearnersGenerating Examples. Mahwah: Lawrence Erlbaum.

Wertsch, J.V . (1991)Voices of the Mind: a sociocultural approach to mediated actio n(Cambridge,Harvard University Press).